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# Describing Function Analysis of Neural Control Vehicle Steering Systems

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## Abstract

In this paper, the robust stability analysis of neural control vehicle steering systems with perturbed parameters is presented. Firstly, the neural controller can be linearized by utilizing the classical describing function. Because the perturbed parameters involving velocity and friction are existed in the vehicle steering system, the stability of the equivalent linearized system is then analyzed by using the parameter plane method. Afterward the amplitude of limit cycles caused by the neural controller can be easily pointed out in the parameter plane. Furthermore, the stability effect with time delay is also addressed in our work. Computer simulation shows the efficiency of this approach.

**Keyword**: Neural, describing function, vehicle, parameter plane

## I. Introduction

Limit cycle prediction of nonlinear control systems has been considered in many academic and industrial applications. The linearizd system based on describing function method has been widely employed in the analysis of nonlinear control system especially when the system has hard nonlinearities like saturation, backlash, hystersis, deadzone and so on [1-6]. For multivariable process control, a method for automatically tuning multivariable PID controllers from relay feedback was proposed in [7]. Chung et al [8] used the describing function method to linearise the nonlinear inductor and estimate the inductance in large current situations. Ackermann and Bunte [9] employed the describing function to predict the limit cycles in the parameter plane of velocity and road tire friction coefficient. The pilot-induced oscillations (PIOs) caused by a complicated interaction between a pilot and vehicle have been considered in [10]. For the intelligent control theory, the describing function technique to design a fuzzy controller for switching DC-DC regulators was proposed in [11]. In addition, the describing functions of neurocontroller have been developed in [12]. In general, the robust analysis of uncertain parameters in linear control system can be dealt with parameter plane method and parameter space method [13, 14]. Due to the results in [12], the main purpose of this paper is to apply the parameter plane method to analyze the stability of a neural control vehicle steering system with perturbed parameters and a time delay for limit cycle prediction.

## II. Preliminaries

In this section, the classical linearized single track vehicle model is given first. The describing function of static neural controller is also introduced. In order to analyze the stability of perturbed parameters, a systematic procedure is proposed to solve this problem by the use of parameter plane method.

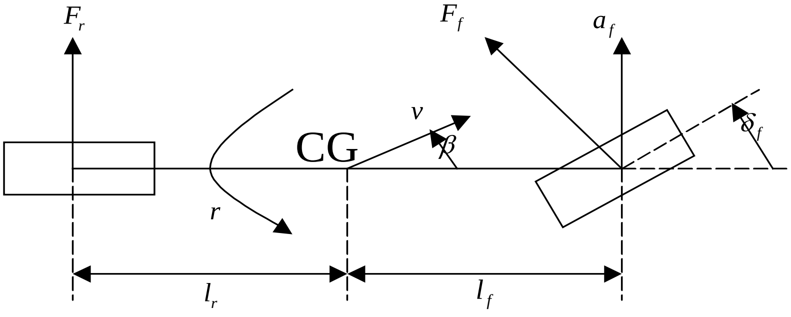

**Fig. 1.** Single track vehicle model

**Table 1.** Vehicle System Quantities

| | |
|---|---|
| $F_f$, $F_r$ | lateral wheel force at front and rear wheel |
| $r$ | yaw rate |
| $\beta$ | side slip angle at center of gravity (CG) |
| $v$ | velocity |
| $a_f$ | lateral acceleration |
| $l_f$, $l_r$ | distance from front and rear axis to CG |
| $l = l_f + l_r$ | wheelbase |
| $\delta_f$ | front wheel steering angle |
| $m$ | mass |

### A. *Description of Vehicle Model [9]*

Fig. 1 shows the single track vehicle model and the related symbols are listed in Table 1. The equations of motion are

$$\begin{bmatrix} mv(\dot{\beta}+r) \\ ml_f l_r \dot{r} \end{bmatrix} = \begin{bmatrix} F_f + F_r \\ F_f l_f - F_r l_r \end{bmatrix}. \tag{1}$$

The tire force can be expressed as

$$F_f(\alpha_f) = \mu c_{f0}\alpha_f, \quad F_r(\alpha_r) = \mu c_{r0}\alpha_r \tag{2}$$

with the tire cornering stiffnesses $c_{f0}, c_{r0}$, the road adhesion factor $\mu$ and the tire side slip angles

$$\alpha_f = \delta_f - (\beta + \frac{l_f}{v} r), \alpha_r = -(\beta - \frac{l_r}{v} r). \tag{3}$$

The state equation of vehicle dynamics with $\beta$ and $r$ can be represented as

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\dfrac{\mu(c_{f0}+c_{r0})}{mv} & -1+\dfrac{\mu(c_{r0}l_r - c_{f0}l_f)}{mv^2} \\ \dfrac{\mu(c_{r0}l_r - c_{f0}l_f)}{ml_f l_r} & -\mu\dfrac{(c_{f0}l_f^2 + c_{r0}l_r^2)}{ml_f l_r v} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \dfrac{\mu c_{f0}}{mv} \\ \dfrac{\mu c_{f0}}{ml_r} \end{bmatrix} \delta_f \tag{4}$$

Hence, the transfer function from $\delta_f$ to $r$ is

$$G_{r/\delta_f} = \frac{c_{f0} m l_f \mu v^2 s + c_{f0} c_{r0} l \mu^2 v}{l_f l_r m^2 v^2 s^2 + l(c_{r0} l_r + c_{f0} l_f) m\mu v s + c_{f0} c_{r0} l^2 \mu^2 + (c_{r0} l_r - c_{f0} l_f) m\mu v^2} \tag{5}$$

The numerical data are listed in Table 2.

**Table 2.** Vehicle System Parameters

| | |
|---|---|
| $c_{f0}$ | 5000 N/rad |
| $c_{r0}$ | 100000 N/rad |
| $m$ | 1830 kg |
| $l_f$ | 1.51 m |
| $l_r$ | 1.32 m |

According to the above analysis of the single track vehicle model, the transfer function from the input of front deflection angle $\delta_f$ to the output of yaw rate $r$ can be obtained as

$$G_{r/\delta_f}(s,\mu,v) = \frac{(1.382\times10^{8}\mu v^{2}s + 1.415\times10^{10}\mu^{2}v)}{6.675\times10^{6}v^{2}s^{2} + 1.08\times10^{9}\mu vs + (1.034\times10^{7}\mu v^{2} + 4\times10^{10}\mu^{2})} \tag{6}$$

The operating range $Q$ of the uncertain parameters $\mu$ and $v$ is depicted in Fig. 2. In addition, the steering actuator is modeled as

$$G_A(s) = \frac{\omega_n^2}{s^2 + \sqrt{2}\omega_n s + \omega_n^2} \tag{7}$$

where $\omega_n = 4\pi$ . In our study, a neural control vehicle steering system with time delay is presented in Fig. 3. The open loop transfer function $G(s)$ is defined as

$$G(s,\mu,v) = \frac{1}{s}G_A(s)G_{r/\delta_f}(s,\mu,v)e^{-sT} \tag{8}$$

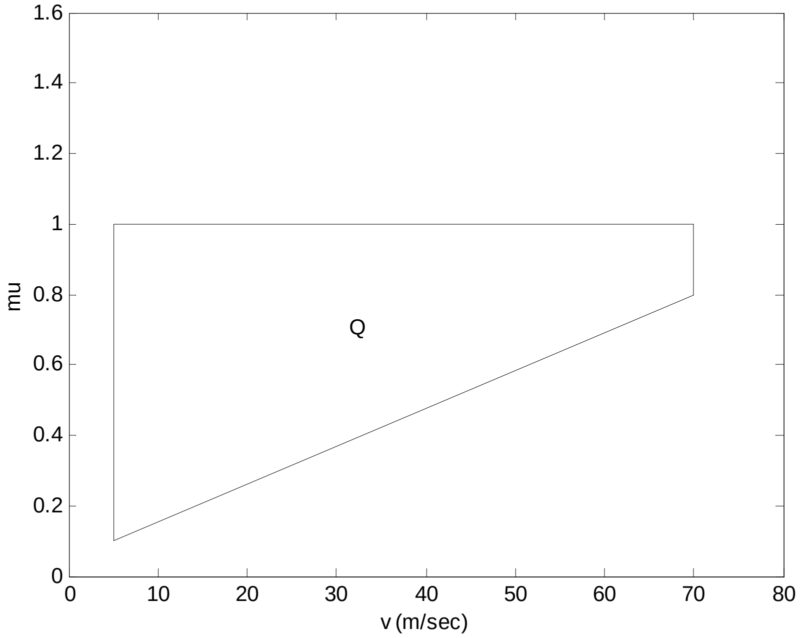

**Fig. 2.** Operating range Q

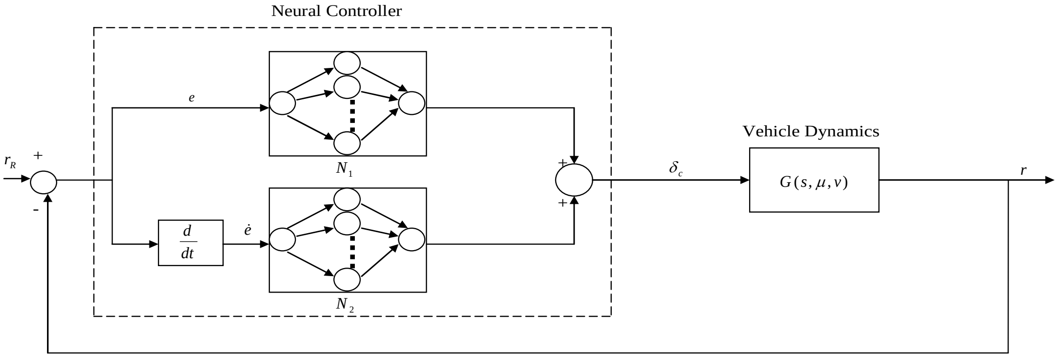

**Fig. 3.** Block diagram of a neural control vehicle steering system

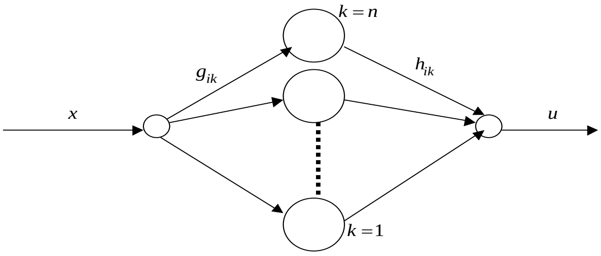

**Fig. 4.** Static neural network

### B. *Describing Functions of Neural Controller and Saturation*

The Static Neural Network (SNN) shown in Fig. 4 can be used as a controller (neural controller) and the input signal is assumed $x_i(t) = A_i \sin \omega t$ . The network structure is 1-n-1 and does not have bias weights [12]. The parameters $g_{ik}$ and $h_{ik}$ are the neural network weights and $n$ is the number of hidden neurons. Based on the stability analysis in [12], the describing function of neurocontroller with sigmoid function tanh may be represented as

$$N_i(A_i) = \sum_{k=1}^{n} g_{ik} \cdot h_{ik} \left\{ 1 - \frac{g_{ik}^2 \cdot A_i^2}{6} \right\} \tag{9}$$

where $A_i$ is the amplitude of limit cycle.

### C. *Parameter Plane Method*

Based on the analysis in the above subsections, the design procedure to analyze the stability of a neural control vehicle steering system with perturbed parameters is given here. From Fig. 3, the closed loop transfer function is

$$\frac{(N_1 + N_2 s)G(s,\mu,v)}{1 + (N_1 + N_2 s)G(s,\mu,v)} = 0 \tag{10}$$

where $N_1$ and $N_2$ are the describing functions of neural controller, respectively.

*Remark 1*. If the input signal of $N_1$ is assumed $e(t) = A_1 \sin \omega t$ where $A_1$ is the amplitude. Then, the amplitude of input signals of $N_2$ can be expressed as $A_2 = A_1 \omega$ .

After some simple manipulations, the characteristic equation can be obtained as

$$f(s,\mu,v,N_1,N_2,T) = C_4\mu^2 + C_3 v^2 + C_2 \mu v + C_1 \mu^2 v + C_0 \mu v^2 = 0 \tag{11}$$

where

$$C_4 = 5.656 \times 10^{10} s(s^2 + 17.7668s + 157.9137),$$

$$C_3 = 9.4384 \times 10^{6} s^3(s^2 + 17.7668s + 157.9137),$$

$$C_2 = 1.5271 \times 10^9 s^2 (s^2 + 17.7668 s + 157.9137),$$

$$C_1 = 2.2345 \times 10^{12} (N_1 + N_2 s) e^{-sT},$$

$$C_0 = 1.4621 \times 10^8 s(s^2 + 17.7688 s + 157.9137) + 2.1818 \times 10^{10} (N_1 + N_2 s) s e^{-sT}.$$

Let $s = j\omega$, (11) is divided into two stability equations with real part $X$ and imaginary part $Y$ of characteristic equation, one has

$$f(j\omega, \mu, v, N_1, N_2, T) = X + jY = 0 \tag{12}$$

where

$$\begin{aligned} X &= -1.0064 \times 10^{12} \omega^2 \mu^2 + 1.6776 \times 10^8 \omega^4 v^2 + (1.5179 \times 10^9 \omega^4 - 2.3999 \times 10^{11} \omega^2) \mu v \\ &\quad + 2.2345 \times 10^{12} (N_1 \cos \omega T + N_2 \omega \sin \omega T) \mu^2 v - (2.5986 \times 10^9 \omega^2 \\ &\quad + 2.1818 \times 10^{10} (N_2 \omega^2 \cos \omega T - N_1 \omega \sin \omega T)) \mu v^2, \\ Y &= (8.9429 \times 10^{12} \omega - 5.656 \times 10^{10} \omega^3) \mu^2 + (9.4399 \times 10^6 \omega^5 - 1.4907 \times 10^9 \omega^3) v^2 \\ &\quad - 2.7008 \times 10^{10} \omega^3 \mu v + 2.2345 \times 10^{12} (N_2 \omega \cos \omega T - N_1 \sin \omega T) \mu^2 v \\ &\quad + (2.3091 \times 10^{10} \omega - 1.4621 \times 10^8 \omega^3 + 2.1818 \times 10^{10} (N_1 \omega \cos \omega T + N_2 \omega^2 \sin \omega T)) \mu v^2. \end{aligned}$$

In order to obtain the solution of $\mu$ and $v$, the following equation is solved

$$\begin{cases} X = 0 \\ Y = 0 \end{cases}, \tag{13}$$

when $N_1$, $N_2$, $T$ are fixed and $\omega$ is changed from 0 to $\infty$. As the amplitude $A_i$ is also changed, the solutions of $\mu$ and $v$ called limit cycle loci can be displayed in the parameter plane. In the following section, two examples are cited to demonstrate the design procedure.

## III. Numerical Results

In our work, three hidden neurons ($n = 3$) of neural network in Fig. 4 are adopted. The weights $g_{ik}$ and $h_{ik}$ are assumed as follows.

$$g_{11} = g_{12} = g_{13} = g_{21} = g_{22} = g_{22} = 1, h_{11} = h_{12} = h_{13} = h_{21} = h_{22} = h_{23} = 0.5.$$

Then, the describing functions $N_1$ and $N_2$ can be obtained by using (9).

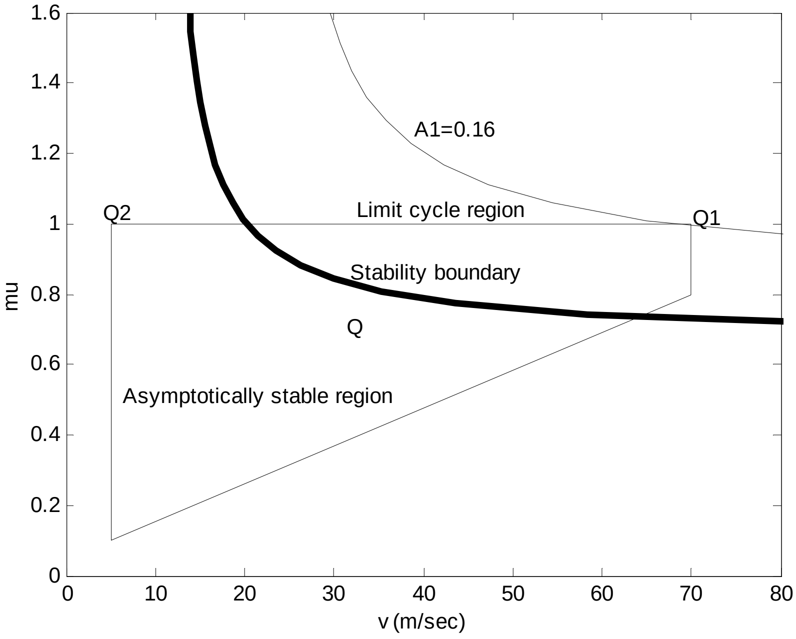

**Fig. 5.** Parameter plane

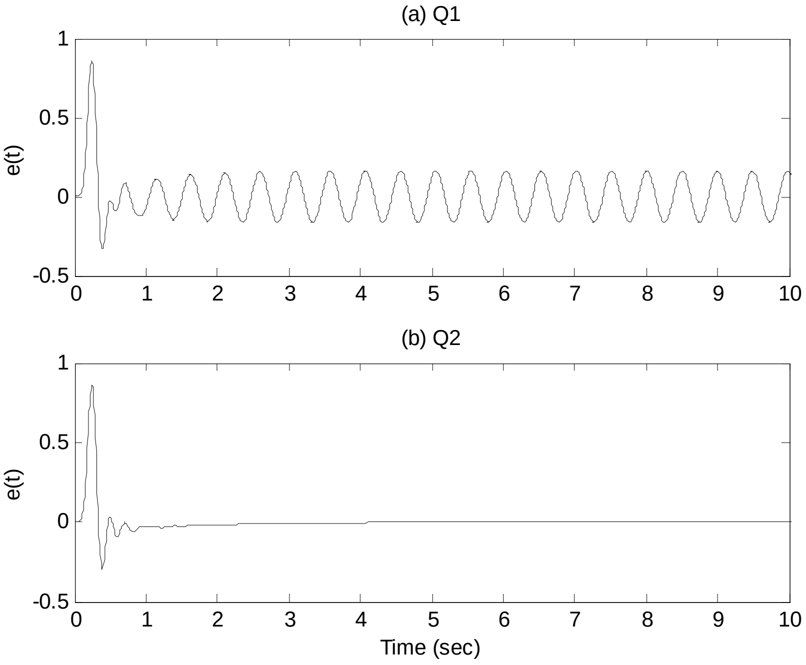

**Fig. 6.** Input signal

### A. *Perturbed Plant Analysis*

Let $T = 0$ first, (13) can be solved when $A$ is fixed and $\omega$ is changed from 0 to $\infty$. Fig. 5 shows the stability boundary and limit cycle locus in $\mu$ - $v$ plane. Two stability regions including asymptotically stable and limit cycle are divided by the stability boundary. In order to test the accuracy of Fig. 5, two operating points Q1 ($\mu = 1$ and $v = 70$) and Q2 ($\mu = 1$ and

$v = 5$) are illustrated. Fig. 6 shows the time responses of input signal $e(t)$. It is obvious that the amplitude of limit cycle shown in Fig. 6 (a) consists with the predicted result ($A = 0.16$) in Fig. 5.

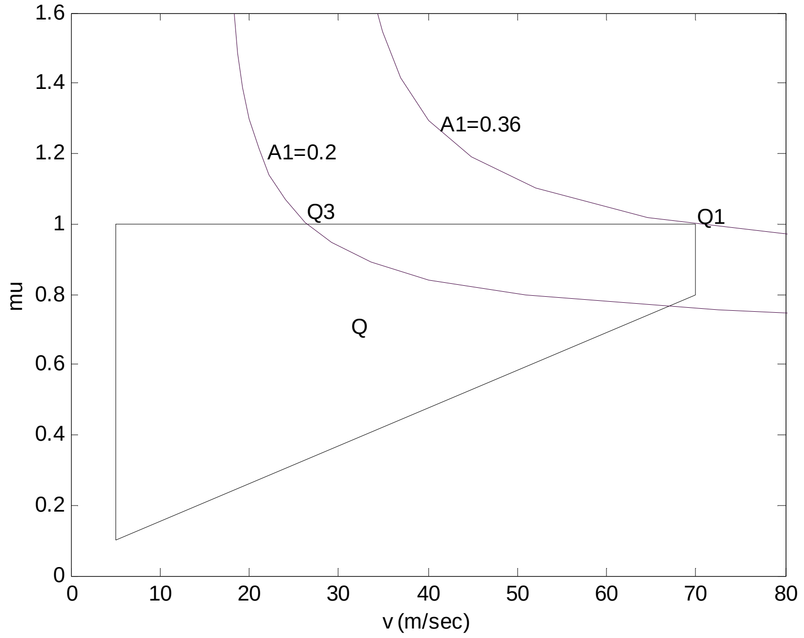

**Fig. 7.** Parameter plane

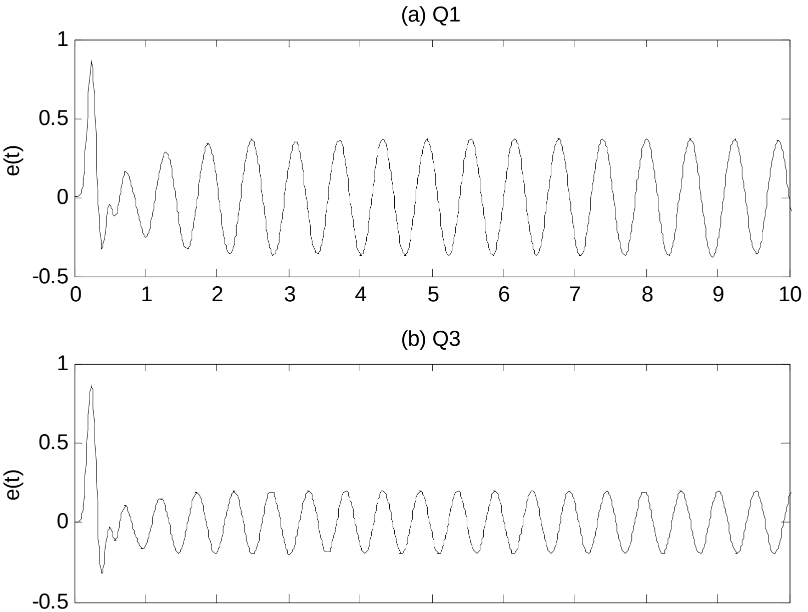

**Fig. 8.** Input signal

### B. *Time Delay Effect*

Let $T = 0.01$, (13) can be solved when $A$ is fixed and $\omega$ is changed from 0 to $\infty$. The weightings of neural controller are the same as before. Fig. 7 shows some limit cycle loci in

$\mu$-$v$ plane. Due to considering the effect of time delay, two operating points Q1 and Q3 ($\mu = 1$ and $v = 25$) are selected. The amplitude of predicted limit cycle Q1 is 0.36 from Fig. 7, which is larger than the result in Fig. 5. It means that if the time delay is existed in the system, the stability margin will be reduced. Fig. 8 shows the time responses of input signal, which matches the result with Fig. 7.

## IV. Conclusion

Based on the approaches of describing function and parameter plane, the stability analysis of neural control vehicle steering systems is proposed in this paper for limit cycle prediction. A simple systematic procedure is presented to deal with this problem. The stability effects of perturbed plant and time delays are both considered. Simulation results show that more information about the characteristic of limit cycle can be obtained by this approach.

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