# Reliability Modeling in Wireless Sensor Networks

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## Abstract

Increasing computing and wireless communication capabilities will expand the role of the sensors from mere information dissemination to more demanding tasks as sensor fusion, classification, collaborative target tracking. Fault tolerance and reliability performs exclusively vital role for embedded systems, such as obscured wireless sensors, which are deployed in some applications where it is difficult to access them physically. This paper is a contributing effort to explore the reliability issues in multifusion sensor networks. We present Markov models of the reliability using different types of sensors and spares that replace sensors when failed. We compare these models in terms of reliability, cost and MTTF (Minimum-Time-To-Fail).

**Keywords**: Fault tolerance, Markov model, multimodal sensor fusion, reliability, wireless sensor networks (WSN).

## I. Introduction

Sensor networks [1],[2].are being used in increasingly diverse applications areas, largely because of their diversity, such as visual, thermal, seismic, acoustic, and magnetic, that can sense (monitor) temperature, noise, physical movement of objects, radioactivity, pressure (air or fluid), and speed. Increasing computing and wireless communication capabilities will expand the role of the sensors from mere information dissemination to more demanding tasks as sensor fusion, classification, collaborative target tracking. Sensor networks do not rely on any hard-wired communication links; therefore, they can be deployed in places without any existing infrastructure, and they can be used in medical assistance, surveillance, reconnaissance, disaster relief operations [3],[4].

The primary idea in this paper is to address and analyze the reliability issues and to device a fault tolerance model in a sensor network system. We emphasize the importance of heterogeneous fault tolerance, where a single type of sensor backs up different types of sensors. We study the redundancy for only one type of sensors to replace only that type, spares to replace any type, and the tradeoff between reliability versus cost. We compare these models in terms of reliability, cost and MTTF (Minimum-Time-To-Fail). Finally we emphasize the similarity between dimension redundancy and error correcting codes. In our models no repair is considered.

## II. Fault Tolerance and Multisensor Fusion

Wireless Sensor Network (WSN) is transforming into a multi-service medium leading to the convergence of voice, video and data communications. Each type of service has a particular constraint and it has to be satisfied for the communication to be effective. For example a voice or video data is delay sensitive and has to be transmitted with in a certain delay. So the service for each and every type of data needs to be met. Traditionally the current infrastructure only provided the best effort service, where the traffic is processed as quickly as possible, but there is no guarantee to the timeliness and assurance of actual delivery. This type of single service can no longer meet the need of the present day constraints. In [5], an interesting research regarding the fault tolerance aspects of a sensor network assumes that the nodes are either active or inactive with Bernoulli model. In case that one or more sensors fail, other sensors of a different type can substitute their work, such that the fault goes undetectable. This is called *multimodal sensor fusion*, and an interesting research of multimodal sensor fusion was done in [6]. The multimodal sensor fusion intrigues scientists in other disciplines, for example a still incompletely solved question is how we identify and deal with three dimensional objects while the eye retina works with only two dimensional patterns of light. Given a network of multitype sensors, we study the aspects of fault tolerance of a multimodal sensor network. We consider different models on achieving fault tolerance. One assumption made of one failure at a time is not a strong assumption; two failures that happen on the same moment can be consider consecutive, because we assume independent events. Another assumption is made is that the failure of the components are independent one another. There are cases of fault dependent events: the temperature raises suddenly, power fluctuations, etc, but we assume that any two faults are independent. As a result, *any two events are disjoint* in term of probabilities.

**Definition** The reliability function of a component at time $t$, $R(t)$ is a conditional probability that the component is operational at time $t$ given that it was operational at time $t_0$. The unreliability of a system is $Q(t) = 1 - R(t)$. For any system, these conditions are generally true:

        - Initially the system is functional at $t=0$: $R(0)=1$, $Q(0) =0$.
        - Eventually the system will fail at $t=$T, $R(T)=0$, $Q(T) =1$.

The *reliability block diagram* (RBD) shows the dependence of the system reliability versus the reliability of each component. The Markov model for reliability of a system is based on two concepts: the possible states of the system, and the transitions between states. The failed state is annotated as F. The *reliability of the system* is defined to be as the probability of the system to be in any of those states except F; it is the probability of being in any state other than F (which is the sum of the probabilities of each state), or 1 – probability of the system to be in the F state. To measure the average time that each system operates before failing we consider the *Minimum-Time-To-Fail (MTTF)*.

**Definition** MTTF is the expected value of the failure time:

$$MTTF = \int_0^\infty t \frac{dQ(t)}{dt} \, dt = \int_0^\infty R(t) dt \,.$$

**Definition** The *failure rate* is defined as the number of failures per time unit:

$$\lambda = \frac{\frac{dR(t)}{dt}}{R(t)} \,.$$

The spares can replace faulty components. We consider in our models *hot* or *stand-by* spares, which means that they replace immediately the failed sensor (there is no gap in time between the moment the sensor has failed and the moment the spare replaces it.) When a spare substitutes a module, then it has the same failure rate as the module. We study different models. We start with a model in which a spare can replace only one type of sensor, so there are different types of spares for different type of sensors, and we consider the case of two types. We continue with spares that can replace any type, and here we consider two-type and three-type pooled spares. To order to achieve a better reliability of the system, one solution is to improve the quality of the spares; another one is to increase the number of spares.

## III.  Modeling Single-type Spares

Let A and B be two different types of sensors and two spares SA and SB that can replace only own type sensors (SA can replace only A, SB only B) (Figure 1).
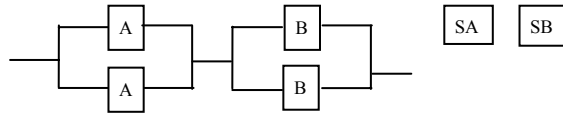


Figure 1. RBD diagram for two-single type spares

Given the failure rate for each component ($\lambda_A, \lambda_B, \lambda_{SA}$, and $\lambda_{SB}$), the Markov model for this example is drawn in Figure 2. If we consider only one spare or no spare, we obtain only portions of the Markov model drawn in Figure 2. If all components have the same failure rate $\lambda$ ($\lambda_A = \lambda_B = \lambda_{SA} = \lambda_{SB} = \lambda$) then the reliability function is

$$R(t) = 9e^{-2\lambda t} - 18e^{-3\lambda t} + 15e^{-4\lambda t} - 6e^{-5\lambda t} + e^{-6\lambda t} \text{ And } MTTF_{two-\sin gle-type} = \frac{73}{60\lambda} = 1.217 .$$

If we consider only one single-type spare, then the reliability function of the system becomes

$$R(t) = 6e^{-2\lambda t} - 9e^{-3\lambda t} + 5e^{-4\lambda t} - e^{-5\lambda t} \text{ And } MTTF_{one-\sin gle-type} = \frac{21}{20\lambda} = 1.05 .$$

If we have no spare, then the reliability function becomes $R(t) = 4e^{-2\lambda t} - 4e^{-3\lambda t} + e^{-4\lambda t}$ and $MTTF_{no-spare} = \frac{11}{12\lambda} = 0.917 .$
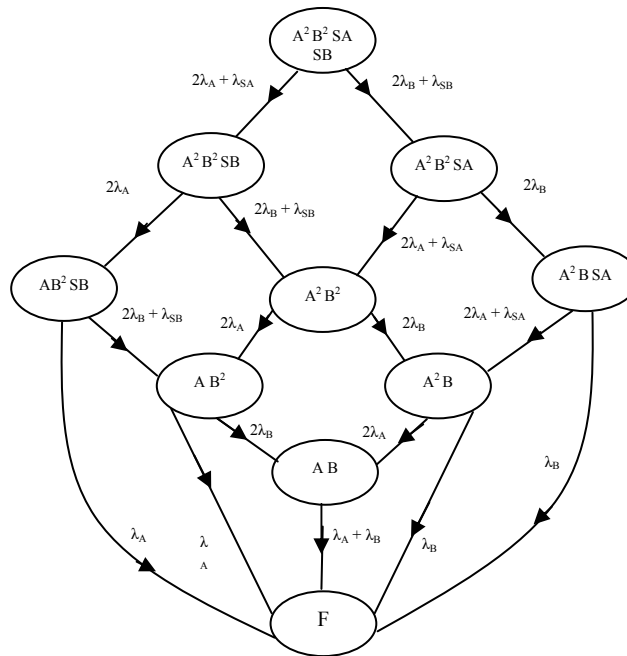


Figure 2. Markov model for two-single type spares

## IV.     Modeling Pooled Spares
Consider the case in which we have pooled spares that can replace any type of sensors.
## Two-type
Let A and B be two different types of sensors, and two spares of type AB that can replace any of the failed sensors, including themselves (see Figure 1 for the same RBD). Given the failure rate for each component ($\lambda_A$, $\lambda_B$, $\lambda_{AB}$) the Markov model is drawn in Figure 3, where S means AB.
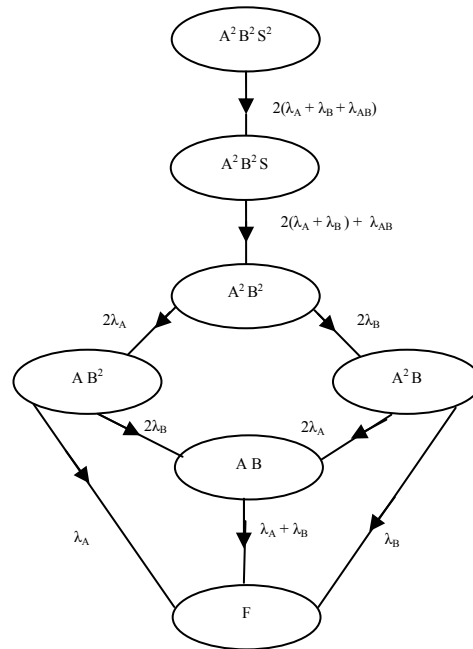
Figure 3. Markov model for two-pooled type spares

Assuming identical failure rates, $\lambda_A = \lambda_B = \lambda_{AB} = \lambda$, the reliability function of the system is

$$R(t) = 10e^{-2\lambda t} - 20e^{-3\lambda t} + 15e^{-4\lambda t} - 4e^{-5\lambda t} \text{ And } MTTF_{two-pooled-type} = \frac{77}{60\lambda} = 1.283.$$

If we consider only one pooled-type spare, then the reliability function becomes

$$R(t) = \frac{20}{3}e^{-2\lambda t} - 10e^{-3\lambda t} + 5e^{-4\lambda t} - \frac{2}{3}e^{-5\lambda t} \text{ And } MTTF_{two-pooled-type} = \frac{67}{60\lambda} = 1.117.$$

If we consider no spare, then

$$R(t) = 4e^{-2\lambda t} - 4e^{-3\lambda t} + e^{-4\lambda t} \text{ And } MTTF_{no-spare} = \frac{11}{12\lambda} = 0.917.$$

### Three-type

Let A, B, and C be three different types of sensors, with the following RBD, and the spare of type ABC can replace any of the failed sensors, including themselves (Figure 4).
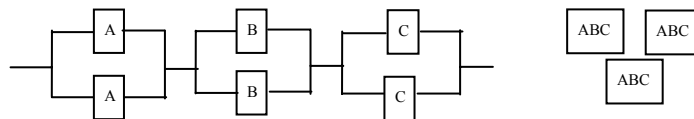


Figure 4. RBD diagram for three-pooled type spares

Given the failure rate for each component, $\lambda_A$, $\lambda_B$, $\lambda_C$, and $\lambda_{ABC}$, the Markov model for this example is drawn in Figure 5, where S means ABC.
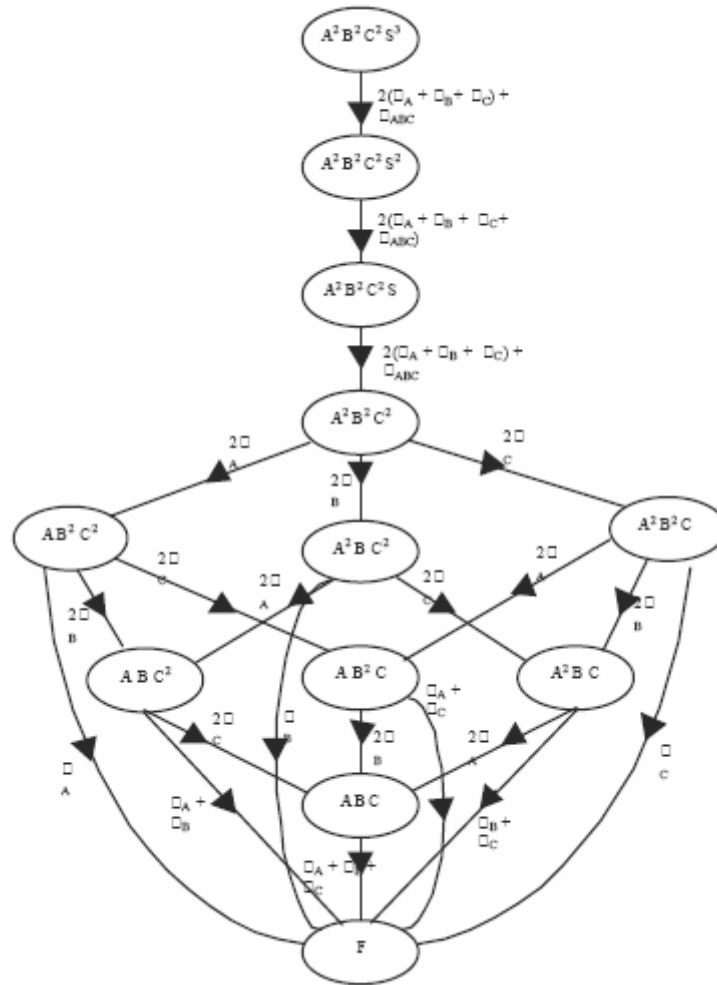
Figure 5. Markov model for three-pooled type spares

If we consider that all units including the spares have the same failure rate λ ($\lambda_A = \lambda_B = \lambda_C = \lambda_{ABC} = \lambda$) then

$$R(t) = \frac{168}{5}e^{-3\lambda t} - \frac{504}{5}e^{-4\lambda t} + 126e^{-5\lambda t} - 84e^{-6\lambda t} + 36e^{-7\lambda t} - \frac{63}{5}e^{-8\lambda t} + \frac{14}{5}e^{-9\lambda t} \quad \text{and}$$

$$MTTF_{three-pooled-type} = \frac{2719}{2520\lambda}.$$

If we consider only two-pooled spares, then the reliability function becomes

$$R(t) = \frac{112}{5}e^{-3\lambda t} - 56e^{-4\lambda t} + 56e^{-5\lambda t} - 28e^{-6\lambda t} + 8e^{-7\lambda t} - \frac{7}{5}e^{-8\lambda t} \quad \text{and}$$

$$MTTF_{three-pooled-type} = \frac{271}{280\lambda}.$$

## V. Reliability versus Cost

Consider the three models: two-single-type spares, two-pooled-type spares and three-pooled-type spares. In Figure 6 are presented different reliability values, taking particular values for $\lambda$: $0.02, 0.03, 0.05$ and $0.10$ as the number of failures per 10000 seconds. Comparing these models in terms of MTTF, the third model has the lowest value, followed by the first model and the second model is the best, independent of the value of $\lambda$:

$$\mathrm{MTTF}_{three-pooled-type} = \frac{2719}{2520\lambda} < \mathrm{MTTF}_{two-\sin gle-type} = \frac{3066}{2520\lambda} < \mathrm{MTTF}_{two-pooled-type} = \frac{3234}{2520\lambda}$$

The cost of a non-redundant system is $C$; the added cost of a simple spare is $c_1$ and the added cost of a pooled spare is $c_2$. If a spare physically replaces a failed sensor then the cost of the system increases from C to $C+c_1$. If a spare virtually replaces a failed sensor, then the cost of the system increases from C to $C+c_2$.
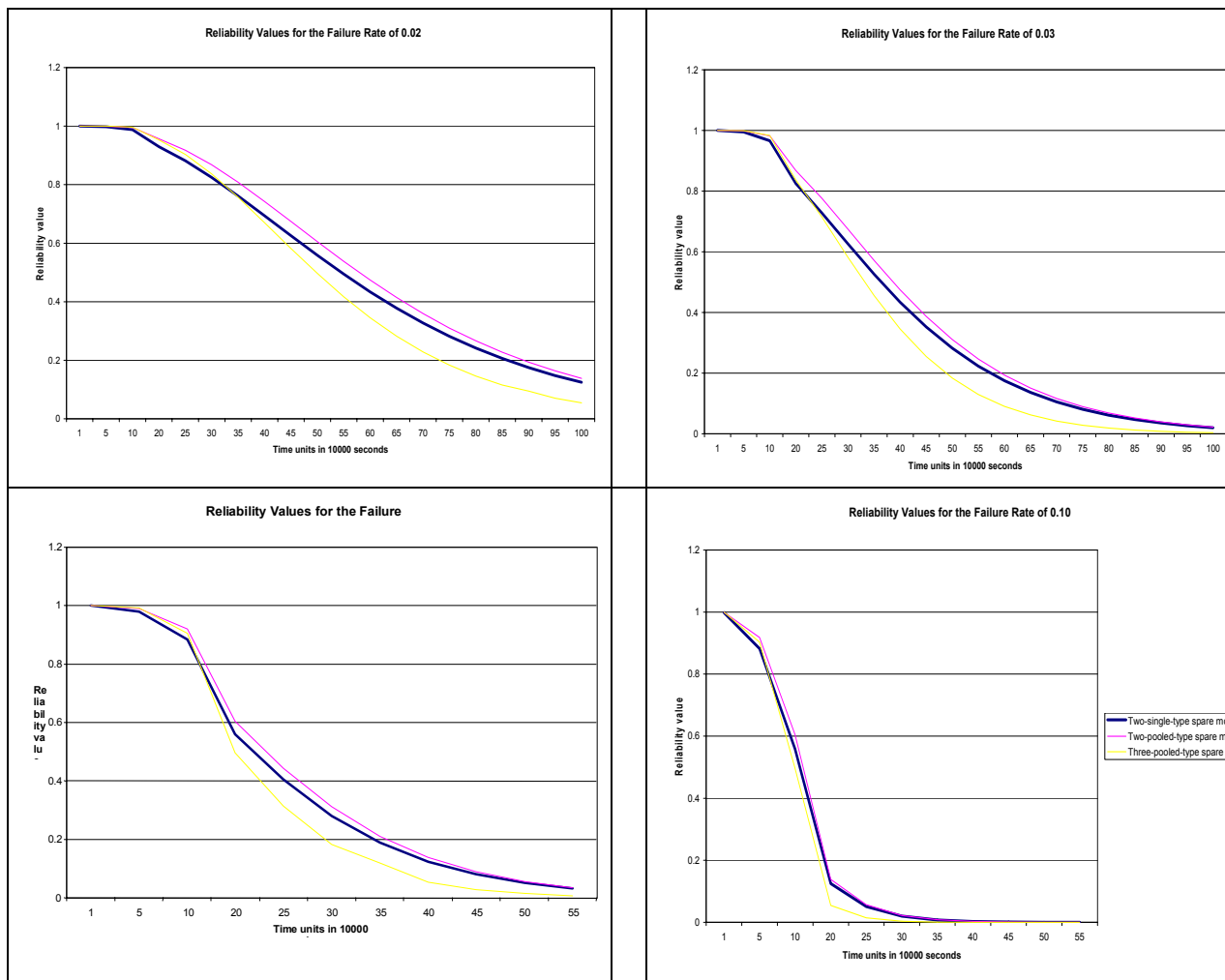


Figure 6. Reliability values for $\lambda = 0.02, 0.03, 0.05$ and $0.10$ failures per 10000 seconds

In Figure 7 an example is shown, where $C=4$ for a two-type sensor system with no redundancy (no spares) with the cost of each component of 0.5. The single-type spare costs the same as one component, $c_1 = 0.5$. The pooled-type spare costs more than one component but less than two components, $c_2 = 0.75$.

## VI. Multifusion Sensor Networks

Consider a set $N$ of $n$ objects ( $|N| = n$ ), a set $M$ of $m$ sensors ( $|M| = m$ ) of $k$ types: $m_i$ sensors of type $i$,

$$\sum_{i=1}^{k} m_i = m .$$ The output of each sensor is binary: $output_i(j) \in \{0,1\}, \forall i \in M, j \in N$ . Shortly, consider $i(j)$ to be $output_i(j)$.

**Definition** Given two objects $a$ and $b$, if $i(a) \neq i(b)$ then we can say that from the point of view of sensor $i$, the objects $a$ and $b$ are *distinguishable*. The assumption of the problem is that no two objects have the same properties, which means that for any two objects, there will be always a sensor which will differentiate them (the outputs of the sensor for those objects will be binary bit-wise).
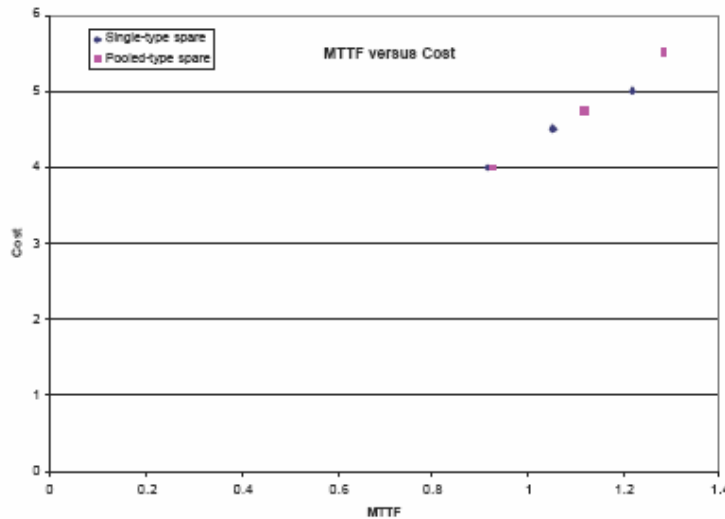


Figure 7. MTTF versus Cost for C=4, $c_1$=0.5 and $c_2$=0.75

**Definition** If $M = \{ s_1, s_2, .. s_m\}$ is a set of sensors, then the *binary coding* of an object $a$ is the ordered set of bits representing the output of each sensor $s_i$ with regard to $a$ : coding $_M(a) = s_1(a) s_2(a)…s_m(a)$.

**Observation 1** By definition, two objects $a$ and $b$ is individualized by the set of sensors $M$ is their binary encodings are different: *coding $_M$ (a) $\neq$ coding $_M$ (b)*.

**Observation 2** The maximum number of sensors required to individualize $n$ objects is $n$-1.

**Observation 3** The minimum number of sensors required to individualize $n$ objects is $\lceil \log n \rceil$.
**Proof** Given $L = \lceil \log n \rceil$, we can have $2^{\lceil \log n \rceil}$ different binary coding of length $L$. Based on observation 1, this means that we can have $2^{\lceil \log n \rceil}$ individualized objects. Because $2^{\lceil \log n \rceil} \geq 2^{\log n} = n$, $L$ is a correct value. We prove by contradiction that taking less than $L$ sensors, we cannot individualize all the $n$ objects. Take $L_1 = \lceil \log n \rceil - 1$ and $M_1$ the set of $L_1$ sensors. The set of all binary coding of length $L_1$ has $2^{L_1}$ elements and

$$\log n + 1 > \lceil \log n \rceil \geq \log n \Rightarrow \log n > \lceil \log n \rceil - 1 \geq \log n - 1 \Rightarrow 2^{\log n} > 2^{\lceil \log n \rceil - 1} \Rightarrow n > 2^{L_1} .$$

So there is only $2^{L_1}$ different binary coding but $n$ objects, which mean that at least two binary coding of the objects in $N$ are the same. This implies that with $L_1$ sensors we cannot individualize each object. As we see from the above observations, there is a parallel between *dimension redundancy* and *error correcting codes*. Given $n$ bits of data, $k$ bits of information and $(n-k)$ redundant bits, out of $2^n$ total number of binary strings, $2^k$ code words can be generated. Without redundant information, you are closing the room for error detection and error correction. So more redundant bits you have, better the chances of getting error correction.

## VII. Conclusions

This paper is a contributing effort to explore the reliability issues in multimodal fusion sensor networks. We presented the system reliability for the case of two types of sensors and three types of sensors. The system reliability is calculated and suggestive values for different λ are given in both cases. We compare these models in terms of reliability, cost and MTTF (Minimum-Time-To-Fail). Finally we emphasize the similarity between dimension redundancy and error correcting codes.

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