D-Stability Analysis of Discrete-Time Perturbed Singular Systems with Multiple Time-Delays*

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Abstract

In this paper, we propose D-stability criteria for discrete-time singular systems with multiple time delays. These criteria guarantee that all the closed-loop generalized eigenvalues of discrete singular systems with multiple time delays are located inside a specific disk $D(\alpha, s)$ centered at $(\alpha, 0)$ with radius s. Then we present sufficient conditions for guaranteeing the D-stability of discrete time-delays singular systems subjected to highly structured parametric perturbations. Using these sufficient conditions, we can estimate the tolerable parametric perturbations bounds that ensure all the closed-loop poles of discrete time-delays perturbed singular systems to remain inside the desired disk $D(\alpha, s)$. These criteria provides efficient tests, easily applied in engineering systems. We give an example illustrating the applicability of the results.

1 Introduction

Singular systems are dynamical systems whose behaviors are governed by both differential equations and algebraic equations. Such systems arise in electrical networks, power systems, large-scale systems, optimal control problems and some population growth models etc. Over the years, the problem of stabilizing time-delay systems has been explored because delays occurs frequently in many engineering systems, such as chemical processes or in long transmission lines or electric networks. Delays, on the other hand, may rise undesirable system responses. Therefore, researches on stability analysis of time-delay systems become essential to practical applications. In designing controlled systems, it is satisfactory in practice to assign all the poles of the closed-loop system in a desired region. A well-known the desired region is for

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discrete systems a disk D centered at $(\alpha,0)$ with radius s, where $s<1$ and $|\alpha|+s<1$. For normal discrete-delay systems, this problem has been studied by Lee and Lee (1986), Furuta and Kim (1987), Tasy *et al.*(1990), Richid (1990), chou *et al.*(1992), Lee *et al.*(1992,1993), Su and Shyr (1994), Wang and Wang(1995), Trinh and Aldeen(1995), and others([14],[15],[17]). However, the problem of D-stability for discrete-time singular systems with multiple time-delays has not been discussed.

In this paper, we introduce the definitions of strongly stability and strongly $D(\alpha,s)$-stability, and propose D-stability criteria for discrete-time singular systems with multiple time delays. The criteria guarantee that all the closed-loop generalized eigenvalues of discrete singular systems with multiple time delays are located inside a specific disk $D(\alpha,s)$ centered at $(\alpha,0)$ with radius s. Then we present sufficient conditions for guaranteeing the D-stability of discrete time-delays singular systems subjected to highly structured parametric perturbations. Moreover, with the sufficient conditions, the allowable perturbations bounds that ensure all the closed-loop poles of discrete time-delays perturbed singular systems to remain inside the desired disk $D(\alpha,s)$ can be estimated. These criteria provides efficient tests, easily applied in engineering systems. An example is given to illustrate the application of the obtained results.

2 Notations and Some Lemmas

Let I_n denote identity matrix of order n. $\rho(A)$ denotes the spectral radius of matrix A. $|A|$ denotes the modulus matrix of A. $j: j=\sqrt{-1}$.

Consider the discrete-time singular system with multiple time-delays as follows,

$$Ex(k+1)=Ax(k)+\sum_{i=1}^{m}B_i x(k-i) \tag{1}$$

where $E, A, B_i \in R^{n\times n}$ are the constant matrices, E is a singular matrix and $rankE=r<n$.

The system (1) can be rewritten as

$$DX(k+1)=\Phi X(k) \tag{2}$$

where $X(k)=[x^T(k), x^T(k-1), \cdots, x^T(k-m)]^T$ and

$$D=\begin{bmatrix} E & & & \\ & I_n & & \\ & & \ddots & \\ & & & I_n \end{bmatrix}, \quad \Phi=\begin{bmatrix} A & B_1 & B_2 & \cdots B_{m-1} & B_m \\ I_n & 0 & 0 & \cdots 0 & 0 \\ 0 & I_n & 0 & \cdots 0 & 0 \\ \vdots & \vdots & \vdots & \cdots \vdots & \vdots \\ 0 & 0 & 0 & \cdots I_n & 0 \end{bmatrix}.$$

If system (2) is regular, then we call system (1) regularity. Without loss of generality, we let

$$E=\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \text{ correspondingly, } A=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \text{ where } A_{11}\in R^{r\times r} \tag{3}$$

In this paper, system (1) is assumed as regular. We first introduced the following Lemmas.
Lemma 1[16]: The necessary and sufficient condition for asymptotic stability of the system (1) is that the every solution z of the generalized characteristic equation ($(\det(zD-\Phi)=0)$ of system (2) Satisfies $|z|<1$, or

$$\det(z^{m+1}E - z^m A - \sum_{i=1}^{m} B_i z^{m-i}) \neq 0 \quad \text{for} \quad |z|\geq 1 \tag{4}$$

Lemma 2[3]: Let $G_1(z)\overset{\Delta}{=}(zI_r - A)^{-1}$, then

$$|G_1(z)| \leq \sum_{k=0}^{\infty} |G_1(k)| \overset{\Delta}{=} L \qquad \text{for} \quad |z|\geq 1$$

where $G_1(k)$ is the pulse-response sequence matrix of $G_1(z)$.

Lemma 3[2]: For any $(n\times n)$ matrix W, the following statement is true.

$\{\rho(W)<1\}$ implies $\{\det(I_n - W)\neq 0\}$.

3 Delays Independent Stability for Discrete-Delays Singular Systems

Definition 1[5]: Let

$$Ex(k+1)=Ax(k) \tag{5}$$

Then (E,A) is said to be strongly stable if $\deg(\det(\lambda E - A)) = rank(E)$, and every generalized eigenvalue λ of system (5) satisfied $|\lambda|<1$.

Theorem 1: If (E,A) is strongly stable, then A_{22} is non-singular and $A_{11}-A_{12}A_{22}^{-1}A_{21}$ is asymptotically stable. Where E,A are as defined in (5), and $A_{11},A_{12},A_{21},A_{22}$ are as defined in (3).

Proof: Since $\det(sE-A)=\det\left(\begin{bmatrix} sI_r - A_{11} & -A_{12} \\ -A_{21} & -A_{22} \end{bmatrix}\right)=\det(-A_{22})s^r + b(s)$

Where $b(s)$ is a polynomial in s , which degree smaller than r. Notice that the assumption:

$\deg(\det(sE - A)) = rank(E) = r$, then $\det(-A_{22}) \neq 0$. Thus A_{22} is nonsingular. By a straightforward calculation, we get

$$\det(sE - A) = \det\left(\begin{bmatrix} sI_r - A_{11} & -A_{12} \\ -A_{21} & -A_{22} \end{bmatrix}\right) = \det(-A_{22})\det(sI_r - (A_{11} - A_{12}A_{22}^{-1}A_{21}))$$

Thus $A_{11} - A_{12}A_{22}^{-1}A_{21}$ is asymptotically stable since by assumption (E, A) is strongly stable.

Let $G(z) = (zE - A)^{-1}, G_1(z) = (zI_r - \hat{A}_{11})^{-1}$,

where, $$\hat{A}_{11} = A_{11} - A_{12}A^{-1}{}_{22}A_{21}. \tag{6}$$

Theorm2: The system (1) is asymptotically stable, if (E, A) is strongly stable, and if the following inequality is satisfied:

$$\rho\{(|V_1||L||V_2| + |V_3|)(|B_1| + |B_2| + \cdots + |B_m|)\} < 1 \tag{7}$$

where

$$V_1 = \begin{bmatrix} I_r \\ -A_{22}^{-1}A_{21} \end{bmatrix},\ \mathrm{V}_2 = \left[I_r, -A_{12}A_{22}^{-1}\right],\quad V_3 = \begin{bmatrix} 0 & 0 \\ 0 & -A_{22}^{-1} \end{bmatrix}$$ and

$$L \stackrel{\Delta}{=} \sum_{k=0}^{\infty} |G_1(k)| \tag{8}$$

where $G_1(k)$ is the pulse-response sequence matrix of the system $G_1(z) = (zI_r - \hat{A}_{11})^{-1}$.

Proof: From Lemma1, a sufficient condition for the asymptotic stability of the system (1) may be expressed as

$$\det[zE - A - \sum_{i=1}^{m} B_i z^{-i}] \neq 0 \qquad \text{for } |z| \geq 1 \tag{9}$$

Since (E, A) is strongly stable, then A_{22} is non-singular and $\hat{A}_{11}$ is asymptotically stable. Thus $\det(zE - A) = \det(-A_{22})\det(zI_r - A_{11} + A_{12}A_{22}^{-1}A_{21}) = \det(-A_{22})\det(zI_r - \hat{A}_{11}) \neq 0$, for $|z| \geq 1$

Then, the inequality (9) can be satisfied if

$$\det[I_n - G(z)\sum_{i=1}^{m} B_i z^{-i}] \neq 0 \qquad \text{for } |z| \geq 1 \tag{10}$$

By Lemma3, the inequality (10) is satisfied if

$$\rho[G(z)\sum_{i=1}^{m} B_i z^{-i}] < 1 \qquad \text{for } |z| \geq 1 \tag{11}$$

Since
$$G(z)=\begin{bmatrix} G_1(z) & -G_1(z)A_{12}A_{22}^{-1} \\ -A_{22}^{-1}A_{21}G_1(z) & -A_{22}^{-1}+A_{22}^{-1}A_{21}G_1(z)A_{12}A_{22}^{-1}\end{bmatrix}$$
$$=\begin{bmatrix} I_r \\ -A_{22}^{-1}A_{21}\end{bmatrix}G_1(z)\begin{bmatrix} I_r & -A_{12}A_{22}^{-1}\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & -A_{22}^{-1}\end{bmatrix}=V_1G_1(z)V_2+V_3$$

Using the property of spectral radius together with Lemma2 and condition (7), it can be show that
$$\rho[G(z)\sum_{i=1}^{m}B_i z^{-i}]\le \rho[|G(z)|\sum_{i=1}^{m}|B_i|\,|z|^{-i}]\le \rho[(|V_1||G_1(z)||V_2|+|V_3|)\sum_{i=1}^{m}|B_i|]$$
$$\le \rho[(|V_1|L|V_2|+|V_3|)\sum_{i=1}^{m}|B_i|]<1$$

for $|z|\ge 1$. This completes the proof of Theorem2.

4 D-Stability Robustness Analysis

4.1 D-stability for discrete-delays singular systems

The problem to be considered in this section is to derive criteria which ensure that all the closed-loop generalized eigenvalues of system (1) are located inside a specified disk $D(\alpha,s)$ centered at $(\alpha,0)$ with radius s as shown in Figure 1, where $|\alpha|+s<1$. Before proceeding to the main results, two definitions must be introduced first.

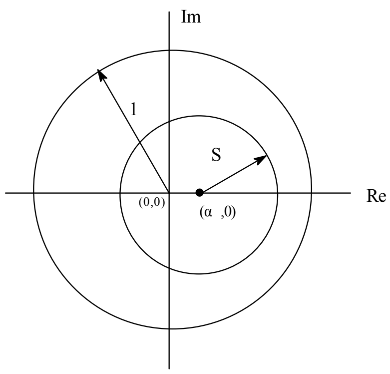

Fig. 1. The specified disk $D(\alpha,s)$.

Definition 2: System (1) is said to be $D(\alpha, s)$-stable, if every generalized eigenvalue z of system (1) satisfied $|(z-\alpha)/s| < 1$, for $s > 0$ and $|\alpha| + s < 1$.

Definition 3: Let

$$Ex(k+1) = Ax(k) \tag{12}$$

Then (E, A) is said to be strongly $D(\alpha, s)$-stable if $\deg(\det(zE - A)) = rank(E)$, and every generalized eigenvalue z of system (12) satisfied $|(z-\alpha)/s| < 1$, for $s > 0$ and $|\alpha| + s < 1$.

Theorem 3: (E, A) is strongly $D(\alpha, s)$-stable if and only if A_{22} is non-singular and all eigenvalues of the matrix $\hat{A}_{11}$ are located inside a specified disk $D(\alpha, s)$.

Where E, A are as defined in (12), and $\hat{A}_{11}$ are as defined in (6).

In the following, we introduce a main theorem. Let

$$\begin{gathered}\overline{G}(w) = (wE - \overline{A})^{-1}, \overline{A} = (A - \alpha E)/s, \overline{A}_{12} = A_{12}/s, \overline{A}_{21} = A_{21}/s, \overline{A}_{22} = A_{22}/s, \\ \overline{V}_1 = \begin{bmatrix} I_r \\ -\overline{A}_{22}^{-1}\overline{A}_{21} \end{bmatrix}, \ \overline{V}_2 = \begin{bmatrix} I_r & -\overline{A}_{12}\overline{A}_{22}^{-1} \end{bmatrix}, \ \overline{V}_3 = \begin{bmatrix} 0 & 0 \\ 0 & -\overline{A}_{22}^{-1} \end{bmatrix}\end{gathered} \tag{13}$$

Theorem 4: The system (1) is $D(\alpha, s)$-stable, if (E, A) is strongly $D(\alpha, s)$-stable, and if the following inequality is satisfied:

$$\rho\{(|\overline{V}_1| L_{\alpha,s} |\overline{V}_2| + |\overline{V}_3|) s^{-1} \sum_{i=1}^{m} \delta |B_i|\} < 1 \tag{14}$$

where δ and $L_{\alpha,s}$ are defined as follows

$$\delta = [\min(|\alpha + s|, |\alpha - s|)]^{-i} \text{ and } L_{\alpha,s} \overset{\Delta}{=} \sum_{k=0}^{\infty} |\overline{G}_1(k)| \tag{15}$$

where $\overline{G}_1(k)$ is the pulse-response sequence matrix of the system $\overline{G}_1(w) = (wI_r - \overline{A}_{11})^{-1}$, with

$\overline{A}_{11} = (\hat{A}_{11} - \alpha I_r)/s$, and $w = (z - \alpha)/s$.

Proof: Based on the proof of Theorem2, a sufficient condition for $D(\alpha, s)$-stability of the system (1) may be expressed as

$$\det[zE - A - \sum_{i=1}^{m} B_i z^{-i}] \neq 0 \qquad \text{for } (|z - \alpha)/s|) \geq 1 \tag{16}$$

Replacing $(z-\alpha)/s$ by w, namely $z = sw + \alpha$, then (16) becomes

$$\det[wE - \overline{A} - s^{-1}\sum_{i=1}^{m} B_i (sw+\alpha)^{-i}] \neq 0 \quad \text{for} \quad |w| \geq 1 \tag{17}$$

Because (E, A) is strongly $D(\alpha, s)$-stable, all eigenvalues of the matrix $\hat{A}_{11}$ are inside the disk $D(\alpha, s)$. It means $\det(wI_r - \overline{A}_{11}) \neq 0$ for $|w| \geq 1$. Then, $\det(wE - \overline{A}) = \det(wI_r - \overline{A}_{11}) \neq 0$
For $|w| \geq 1$. Then, the inequality (17) can be satisfied if

$$\det[I_n - \overline{G}(z)s^{-1}\sum_{i=1}^{m} B_i (sw+\alpha)^{-i}] \neq 0 \qquad \text{for } |w| \geq 1 \tag{18}$$

By Lemma 3, the inequality (18) is satisfied if

$$\rho[\overline{G}(z)s^{-1}\sum_{i=1}^{m} B_i (sw+\alpha)^{-i}] < 1 \qquad \text{for } |w| \geq 1 \tag{19}$$

Since

$$\overline{G}(z) = \begin{bmatrix} \overline{G}_1(z) & -\overline{G}_1(z)\overline{A}_{12}\overline{A}_{22}^{-1} \\ -\overline{A}_{22}^{-1}\overline{A}_{21}\overline{G}_1(z) & -\overline{A}_{22}^{-1} + \overline{A}_{22}^{-1}\overline{A}_{21}\overline{G}_1(z)\overline{A}_{12}\overline{A}_{22}^{-1} \end{bmatrix}$$
$$= \begin{bmatrix} I_r \\ -\overline{A}_{22}^{-1}\overline{A}_{21} \end{bmatrix} \overline{G}_1(z)\begin{bmatrix} I_r & -\overline{A}_{12}\overline{A}_{22}^{-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -\overline{A}_{22}^{-1} \end{bmatrix} = \overline{V}_1\overline{G}_1(z)\overline{V}_2 + \overline{V}_3$$

Using the property of spectral radius together with Lemma2 and condition(14), it can be show that

$$\rho[\overline{G}(z)s^{-1}\sum_{i=1}^{m} B_i (sw+\alpha)^{-i}] \leq \rho[|\overline{G}(z)|\, s^{-1}\sum_{i=1}^{m}(|sw+\alpha|^{-i}|B_i|)]$$
$$\leq \rho[(|\overline{V}_1||\overline{G}_1(z)||\overline{V}_2| + |\overline{V}_3|)s^{-1}\sum_{i=1}^{m}\delta|B_i|] \leq \rho[(|\overline{V}_1|L_{\alpha,s}|\overline{V}_2| + |\overline{V}_3|)s^{-1}\sum_{i=1}^{m}\delta|B_i|] < 1$$

for $|w| \geq 1$. This completes the proof of Theorem 4.

3.2 D-stability robustness for discrete-delays perturbed singular systems

Consider the following discrete-delays singular system with parametric perturbations:

$$Ex(k+1) = (A + \Delta A)x(k) + \sum_{i=1}^{m}(B_i + \Delta B_i)x(k-i) \tag{20}$$

where $E, A, B_i \in R^{n\times n}$ are the constant matrices, E is a singular matrix and $rankE = r < n$. ΔA, ΔB_i are $n\times n$ perturbed matrices with

$$|\Delta A| \le U, |\Delta B| \le U_i \quad (i = 1,2,\cdots,m) \tag{21}$$

where U, U_i are nonnegative matrices representing highly structured parametric perturbations.

Theorem 5: All generalized eigenvalues of the perturbed discrete time-delays singular system (20) remain inside the specified disk $D(\alpha, s)$, if (E, A) is strongly $D(\alpha, s)$-stable, and if the following inequality is satisfied:

$$\rho\{(|\overline{V}_1| L_{\alpha,s} |\overline{V}_2| + |\overline{V}_3|)s^{-1}[U + \sum_{i=1}^{m}\delta(|B_i| + U_i)]\} < 1 \tag{22}$$

Where $\overline{V}_1, \overline{V}_2, \overline{V}_3$ are as defined in (13), and $\delta, L_{\alpha,s}$ are as defined in (15).

Proof: From the proof of Theorem2, a sufficient condition for $D(\alpha, s)$-stability of the perturbed system (20) may be expressed as

$$\det[zE - (A + \Delta A) - \sum_{i=1}^{m}(B_i + \Delta B_i)z^{-i}] \ne 0 \qquad \text{for } (|z-\alpha)/s| \ge 1 \tag{23}$$

Replacing $(z-\alpha)/s$ by w, following in the same lines as the proof of Theorem 4, (23) becomes

$$\det[wE - \overline{A} - s^{-1}\Delta A - s^{-1}\sum_{i=1}^{m}(B_i + \Delta B_i)(sw+\alpha)^{-i}] \ne 0 \quad \text{for } |w| \ge 1 \tag{24}$$

Since (E, A) is strongly $D(\alpha, s)$-stable, then all eigenvalues of the matrix $\hat{A}_{11}$ are inside the disk $D(\alpha, s)$. It means $\det(wI_r - \overline{A}_{11}) \ne 0$ for $|w| \ge 1$. Then, $\det(wE - \overline{A}) = \det(wI_r - \overline{A}_{11}) \ne 0$

For $|w| \ge 1$. Then, the inequality (24) can be satisfied if

$$\det\{I_n - \overline{G}(z)s^{-1}[\Delta A + \sum_{i=1}^{m}(B_i + \Delta B_i)(sw+\alpha)^{-i}]\} \ne 0 \quad \text{for } |w| \ge 1 \tag{25}$$

By Lemma 3, the inequality (25) is satisfied if

$$\rho\{\overline{G}(z)s^{-1}[\Delta A + \sum_{i=1}^{m}(B_i + \Delta B_i)(sw+\alpha)^{-i}]\} < 1 \qquad \text{for } |w| \ge 1 \tag{26}$$

Since

$$\overline{G}(z)=\begin{bmatrix}\overline{G}_1(z) & -\overline{G}_1(z)\overline{A}_{12}\overline{A}_{22}^{-1}\\ -\overline{A}_{22}^{-1}\overline{A}_{21}\overline{G}_1(z) & -\overline{A}_{22}^{-1}+\overline{A}_{22}^{-1}\overline{A}_{21}\overline{G}_1(z)\overline{A}_{12}\overline{A}_{22}^{-1}\end{bmatrix}$$
$$=\begin{bmatrix}I_r\\ -\overline{A}_{22}^{-1}\overline{A}_{21}\end{bmatrix}\overline{G}_1(z)\begin{bmatrix}I_r & -\overline{A}_{12}\overline{A}_{22}^{-1}\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & -\overline{A}_{22}^{-1}\end{bmatrix}=\overline{V}_1\overline{G}_1(z)\overline{V}_2+\overline{V}_3$$

Using the property of spectral radius together with Lemma2 and condition (22), it can be show that

$$\rho\{\overline{G}(z)s^{-1}[\Delta A+\sum_{i=1}^{m}(B_i+\Delta B_i)(sw+\alpha)^{-i}]\}\le\rho\{|\overline{G}(z)|\,s^{-1}[|\Delta A|$$
$$+\sum_{i=1}^{m}(|sw+\alpha|^{-i}|B_i+\Delta B_i|)]\}<1$$

for $|w|\ge 1$. This completes the proof of Theorem 5.

Remark 2: $L_{\alpha,s}$ can be obtained from (15), where $\overline{G}_1(k)=\overline{A}_{11}^{k-1}, k=1,2,\cdots,\infty,\ \ \overline{G}_1(0)=0$. Since $\overline{A}$ is stable, the convergence of the $\sum_{k=0}^{\infty}|\overline{G}_1(k)|$ is ensured.

5 Illustrative Examples

Let a discrete-delays singular system include parametric perturbations as described by the following equation:

$$Ex(k+1)=(A+\Delta A)x(k)+\sum_{i=1}^{2}(B_i+\Delta B_i)x(k-i) \tag{27}$$

where

$$E=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix},\quad A=\begin{bmatrix}0.40 & 0.000 & 0.10\\ -0.2 & 0.506 & -0.03\\ 0.00 & -0.10 & 0.50\end{bmatrix},\quad B_1=\begin{bmatrix}0.0012 & 0.0013 & 0.0002\\ -0.0024 & 0.0015 & 0.0016\\ -0.0008 & 0.0033 & -0.0027\end{bmatrix},$$

$B_2=\begin{bmatrix}0.0002 & -0.0006 & 0.0004\\ 0.0000 & -0.0005 & -0.0003\\ 0.0008 & 0.0001 & -0.0007\end{bmatrix}$. By calculating, $\hat{A}_{11}=\begin{bmatrix}0.40 & 0.02\\ -0.2 & 0.5\end{bmatrix}$. The eigenvalues of $\hat{A}_{11}$ are $0.45\pm 0.0387j$. Without loss of generality, let $\alpha=0.1, s=0.5$, thus, $\hat{A}_{11}$ is $D(0.1,0.5)$-stable, and A_{22}

degenerates a non-zero constant. Thus, (E, A) is strongly $D(0.1,0.5)$ -stable. $L_{\alpha,s} = \sum_{k=1}^{\infty} |\overline{G}_1(k)| = \sum_{k=1}^{\infty} |\overline{A}_{11}|^{k-1} = \begin{bmatrix} 25/8 & 5/8 \\ 25/4 & 25/4 \end{bmatrix}$. If we set

$$|\Delta A| \le U = \begin{bmatrix} 0.0011 & 0.0014 & 0.0000 \\ 0.0012 & 0.0013 & 0.0018 \\ 0.0013 & 0.0015 & 0.0016 \end{bmatrix}, |\Delta B_1| \le U_1 = \begin{bmatrix} 0.0012 & 0.0011 & 0.0000 \\ 0.0011 & 0.0013 & 0.0017 \\ 0.0013 & 0.0015 & 0.0014 \end{bmatrix},$$

$$|\Delta B_2| \le U_2 = \begin{bmatrix} 0.0002 & 0.0006 & 0.0001 \\ 0.0001 & 0.0004 & 0.0003 \\ 0.0005 & 0.0001 & 0.0002 \end{bmatrix}, \text{Then,}$$

$$\rho\{(|\overline{V}_1| L_{\alpha,s} |\overline{V}_2| + |\overline{V}_3|) s^{-1} [U + \sum_{i=1}^{2} \delta(|B_i| + U_i)]\} = 0.5777 < 1.$$

Therefore, all generalized eigenvalues of the perturbed discrete time-delays singular system (27) remain inside the specified disk $D(0.1,0.5)$.

6 Conclusions

In this paper, we have established the sufficient criteria of delays independent asymptotic stability and robust D-stability for perturbed discrete-time singular systems with multiple time-delays. We have estimated the tolerable parametric perturbation bounds for guaranteeing that all generalized eigenvalues of discrete time-delays perturbed singular systems to inside the desired disk $D(\alpha, s)$, and providing both good stability robustness and effective performance robustness effective measures both stability robustness and performance robustness. It is obvious that these criteria are easy test and convenient for the application in the practice. Furthermore, An illustrative is given to illustrate the application of the obtained result.

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