Robust Fault-tolerant Controller Design for Linear Uncertain Systems with Multiple Time delays

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Abstract

In this paper, based on Riccati's equation and Lypunove's stability, the problem of robust fault tolerant control laws for a type of linear uncertain systems with multiple time-delays is studied. The output feedback with multiple time-delays is put forward and the sufficient conditions of integrity for actuator and sensor failures are proposed. Finally, simulation is given to demonstrate the effectiveness of the proposed approach and better stability criterion can be obtained compared to the case of output feedback without time-delay.

Keyword: Fault-tolerant controller, Linear uncertain systems with multiple time-delays, Integrity design, Robustness.

I. Introduction

Modern technological systems rely heavily on sophisticated control systems to meet increased safety and performance requirements. This is particularly true in safety critical application, such as aircraft, spacecraft, nuclear power plants, and chemical plants processing hazardous materials, where a minor and often benign fault could potentially develop into catastrophic events if left unattended for or incorrectly responded to. So the growing demand for reliability, maintainability and survivability has been higher and higher. To prevent fault induced losses and to minimize the potential risks, new control techniques and design approaches need to be developed to cope with system component malfunctions whilst maintaining the desirable degree of overall system stability and performance levels. A control system that possesses such a capability is often known as a fault-tolerant control system (FTCS)[1].

The main object of fault-tolerant control (FTC) technology is to guarantee the safety and satisfactory behavior of closed-loop systems under various types of failures. Depending on how the redundancy is being utilized, existing efforts in FTCS design can be classified into two main approaches： passive FTC and active FTC. In a passive approach, the conceivable system component failures are assumed to be known a priori, and the control system takes into account of all these failure modes in the design stage. Once the control system is designed, it will remain fixed during the system operation. Even in the event of component failures, the control system should still be able to maintain the designed performance. In contrast, an active FTCS reacts to the system component failures actively by properly reconfiguring its control actions so that the stability of the entire system can still be acceptable. To achieve a successful control system reconfiguration, this approach relies heavily on a real-time fault detection/diagnosis scheme for the most up-to-date information about the status of the system and the operating conditions of its components. Design of integrity is a type of passive FTCS. For an MIMO system, sensor or actuator failures in some feedback channels may result in unsatisfactory system performance, or even instability. It is therefore of interest to develop controllers with integrity, which maintain the stability of closed-loop systems despite sensor and actuator failures.

There have been many results on FTC with integrity. Shimemura and Fujita introduced a "switching matrix" to represent sensor or actuator failures, and utilized a positive semi-definite solution of a Ricatti equation to determine a state feedback control which satisfies both the response and integrity requirements. In the frequency domain, the same authors proved that integrity is ensured if and only if all principal minors of the return difference matrix or the sensitivity matrix are units, with the controller being stable. Recently, a number of research works were focused on integrity design and got great achievement [2-16]. However, those methods generally use state feedback and do not consider the uncertainty and multiple time delays at the same time. Even using the output feedback controller, the controller with delay is not considered. As we all know, some states of a real system are not accessible completely. So the application area of integrity design with state feedback is limited. Moreover, In fact, a lot of real systems have multiple time delays and uncertain because of transmission time delay, modeling error and so on. So it is important to study integrity design for linear uncertain systems with multiple time delays based on output feedback with multiple time-delays.

In order to consider the uncertainty and multiple time delays at the same time, an output feedback controller with multiple time delays for a type of linear uncertain systems with multiple time delays is put forward based on the Riccati function and Lyapunov stability theorem. The paper is organized into the following section. System description is described in section2. The lemma, the design of fault-tolerant controller with multiple time-delays and the design step are described in section 3. At last, a simulation shows that the design method of fault-tolerant controller possesses integrity against actuator and sensor failures.

II. Problem Description

Consider the following linear uncertain system with multiple time delays

$$\begin{cases} \dot{x}(t) = (A_0 + \Delta A_0(\mathrm{t}))x(t) + \sum_{i=1}^{q}(A_i + \Delta A_i(t))x(t-h_i) + (B+\Delta B(t))u(t) \\ y(t) = (C+\Delta C(t))x(t) \end{cases} \tag{1}$$

where $A_i \in R^{n\times n}(i=0,1,2,...,q), B \in R^{n\times m}$ and $C \in R^{n\times n}$ are continuous matrices of appropriate dimensions, $\Delta A_i(t) \in R^{n\times n}(i=0,1,2,...,q), \Delta B(t) \in R^{n\times m}$ and $\Delta C(t) \in R^{n\times n}$ denote the parameter uncertainties. $x(t) \in R^n$ and $u(t) \in R^m$ are respectively state vector and input vector. $y(t) \in R^n$ is output vector and $h_i > 0$ is time delay. $x(t_0) = \varphi(t_0)$ is initial condition of system ($t_0 \in [-h_i, 0)$, $\varphi(t_0)$ is continuous function in $[-h_i, 0]$).

Supposed that $(A_i, B)(i = 1,2,...,q)$ is controllable, the output feedback controller with multiple time delays can be displayed as follow:

$$u(t) = Ky(t) + \sum_{i=1}^{q} F_i y(t - h_i) \tag{2}$$

The corresponding state function of closed-loop system is as follow

$$\begin{aligned}\dot{x}(t) = (A_0 + \Delta A_0(t) + (B + \Delta B(t))K(C + \Delta C(t)))x(t) + \\ \sum_{i=1}^{q} (A_i + \Delta A_i(t) + (B + \Delta B(t))F_i(C + \Delta C(t)))x(t - h_i)\end{aligned} \tag{3}$$

How to ascertain F_i ? Ref [17] indicates that the poles of $(A_i + BF_iC)(i = 1,...,q)$ are collocated arbitrarily by selecting F_i if $(A_i, B)(i = 1,2,...,q)$ is controllable. That is to say, module of $(A_i + BF_iC)$ can be reduced arbitrarily by selecting F_i. Moreover, the effect of time delay can be reduced for that closed loop systems. Of course, F_i has a lot of choices and determinate DOF. Theorems that are proposed by the author can make linear uncertain systems with multiple time delays possess integrity for actuator and sensor failures.

III. Main Result

Lemma[18] Suppose that X and Y are matrices with appropriate dimension and γ is a random positive real number, we can get the following inequation :

$$X^T Y + Y^T X \le \gamma X^T X + \gamma^{-1} Y^T Y$$

3.1 Fault-tolerant Controller Design against Actuator Failures

Suppose that actuator failures has $N(\le 2^m)$ modes, namely $L = \{L_0, L_1, \cdots, L_N\}$.

where $L_i = diag(l_1, l_2, \cdots, l_m)$, $l_i = \begin{cases} 1 & no \quad actuator \quad fault \\ 0 & ith \quad actuator \quad fault \end{cases}$.

The corresponding fault system can be displayed as follow:

$$\begin{cases} \dot{x}(t) = (A_0 + \Delta A_0(t))x(t) + \sum_{i=1}^{q} (A_i + \Delta A_i(t))x(t - h_i) + (B + \Delta B(t))L_j u(t) \\ y(t) = (C + \Delta C(t))x(t) \end{cases} \tag{4}$$

And then combining the formula (2) and (4), we can get the following closed-loop system with actuator faults:

$$\begin{aligned}\dot{x}(t) &= (A_0 + \Delta A_0(t) + (B + \Delta B(t))L_j K(C + \Delta C(t)))x(t) + \\ &\sum_{i=1}^{q}(A_i + \Delta A_i(t) + (B + \Delta B(t))L_j F_i(C + \Delta C(t)))x(t - h_i)\end{aligned} \tag{5}$$

Theorem1. Suppose that the uncertain parameters of linear uncertain system with multiple time delays (4) satisfy

$$\| \Delta A_i \| \le a_i (i = 0,1,2,...,q), \| \Delta B \| \le b, \| \Delta C \| \le c$$

where $\| \bullet \|$ denotes the norm. $a_i (i = 0,1,2,...,q), b$ and c are certainly fluctuant norm limit of model matrices.

If we use output feedback controller with multiple time delays (7) and the following condition (6) can be retained

$$\min_{L_i \in L} \lambda_m (T^j) > 0 \tag{6}$$

Then Linear uncertain closed-loop system with multiple time delays (5) possessing fault-tolerant for actuator failures $L_j \in L$ is given.

$$u(t) = Ky(t) + \sum_{i=1}^{q} F_i y(t - h_i) = -R^{-1} B^T Py(t) + \sum_{i=1}^{q} F_i y(t - h_i) \tag{7}$$

where P is positive definite solution of the following algebraic Riccati equation.

$$\begin{cases} A_0{}^T P + PA_0 - PBR^{-1} B^T PC - C^T PBR^{-1} B^T P + 2\alpha P + Q = 0 \\ \alpha \ge 0, Q \ge 0, R \ge 0 \end{cases} \tag{8}$$

$\lambda_m (T^j)$ denotes the minimal eigenvalue of the following real symmetrical matrix.

$$\begin{aligned} T^j &= -(BL_j KC)^T P - PBL_j KC + Q + 2(\alpha - a_0)P + C^T PBK \\ &- bcK^T L_j{}^T P - bC^T K^T L_j{}^T P - cK^T L_j{}^T B^T P + PBKC - bcPL_j K \\ &- \sum_{i=1}^{q} \gamma_i{}^{-1} I_n - bPL_j KC - cPBL_j K - \sum_{i=1}^{q} \gamma_i D_j{}^T PE_j E_j{}^T PD_j \end{aligned} \tag{9}$$

where
$$\begin{aligned} K &= -R^{-1} B^T P \\ D_j &= A_0 + a_0 I_n + (B + bI_{n\times m})L_j K(C + cI_n) \\ E_j &= A_i + a_i I_n + (B + bI_{n\times m})L_j F_i (C + cI_n) \end{aligned}$$

Proof Define a Lyapunov function as

$$V(x,h) = x^T(t)Px(t) + \sum_{i=1}^{q} \gamma_i{}^{-1} \int_{t-h_i}^{t} x^T(\tau)x(\tau)d\tau \tag{10}$$

where P is positive definite solution of the algebraic Riccati equation(8), $\gamma > 0$. We get the differential coefficient of $V(x,t)$ against t along state trajectory ascertained by state function (5):

$$\dot{V}(x,h) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + \sum_{i-1}^{q}\gamma_i^{-1}x^T(t)x(t) - \sum_{i=1}^{q}\gamma_i^{-1}x^T(t-h)x(t-h) = [(A_0 + \Delta A_0(t) + (B+\Delta B(t))L_jK(C+\Delta C(t)))x(t) + \sum_{i=1}^{q}(A_i + \Delta A_i(t) + (B+\Delta B(t))L_jF_i(C+\Delta C(t)))x(t-h)]^T Px(t) + x^T(t)P[(A_0 + \Delta A_0(t) + (B+\Delta B(t))L_jK(C+\Delta C(t)))x(t) + \sum_{i=1}^{q}(A_i + \Delta A_i(t) + (B+\Delta B(t))L_jF_i(C + \Delta C(t)))x(t-h)] + \sum_{i=1}^{q}\gamma_i^{-1}x^T(t)x(t) - \sum_{i=1}^{q}\gamma_i^{-1}x^T(t-h)x(t-h) \le x^T(t)\{[A_0 + \Delta A_0(t) + (B+\Delta B(t)) L_jK(C+\Delta C(t))]^T P + P[A_0 + \Delta A_0(t) + (B+\Delta B(t))L_jK(C+\Delta C(t))] + \sum_{I=1}^{Q}\gamma_i^{-1}I_n + \sum_{i=1}^{q}\gamma_i[A_0 + \Delta A_0(t) + (B+\Delta B(t))L_jK(C+\Delta C(t))]^T P[A_i + \Delta A_i(t) + (B+\Delta B(t))L_jF_i(C+\Delta C(t))][A_i + \Delta A_i(t) + (B + \Delta B(t))L_jF_i(C+\Delta C(t))]^T P[A_0 + \Delta A_0(t) + (B+\Delta B(t))L_iK(C+\Delta C(t))]\}x(t) \le x^T(t)\{[A_0 + a_0I_n + (B+bI_{n\times m})L_iK(C+cI_n)]^T P + P[A_0 + a_0I_n + (B+bI_{n\times m})L_iK(C+cI_n)] + \sum_{i=1}^{q}\gamma_i^{-1}I_n + \sum_{i=1}^{q}\gamma_i[A_0 + a_0 I_n + (B+bI_{n\times m})L_iK(C+cI_n)]^T P[A_i + a_iI_n + (B+bI_{n\times m})L_jF_i(C+cI_n)][A_i + a_iI_n + (B+bI_{n\times m})L_j F_i(C+cI_n)]^T P[A_0 + a_0I_n + (B+bI_{n\times m})L_jK(C+cI_n)]\}x(t) = -x^T(t)[-(BL_jKC)^T P - PBL_jKC + Q + 2(\alpha - a_0)P + C^TPBK - bcK^TL_j^TP - bC^TK^TL_j^TP - cK^TL_j^TB^TP + PBKC - bcPL_jK - \sum_{i=1}^{q}\gamma_i^{-1}I_n - bPL_jKC - cPBL_jK - \sum_{i=1}^{q}\gamma_iD_i^TPE_iE_i^TPD_i]x(t) = -x^T(t)T^jx(t)$$

$$\le -\lambda_m(T^j)x^T(t)x(t)$$

As we all know, when condition (6) is held, $\dot{V}(x,h)$ will be smaller than zero. Based on the Lyapunov theorem, stability of the closed-loop system (5) is retained. That is to say, output feedback controller with multiple time delays (7) has fault-tolerant for actuator failure $L_i \in L$.

3.2 Fault-tolerant Controller Design against Sensor Failures

Suppose that sensor failures has $N(\le 2^p)$ modes, namely $M = \{M_0, M_1, \cdots, M_N\}$.

where $M_i = diag(m_1, m_2, \cdots, m_n)$, $m_i = \begin{cases} 1 & no \quad sensor \quad fault \\ 0 & ith \quad sensor \quad fault \end{cases}$.

The corresponding fault system can be displayed as follow：

$$\begin{cases} \dot{x}(t) = (A_0 + \Delta A_0(t)x(t) + \sum_{i=1}^{q}(A_i + \Delta A_i(t))x(t-h_i) + (B+\Delta B(t))u(t) \\ y(t) = M_j(C+\Delta C(t))x(t) \end{cases} \tag{11}$$

Combine the (2) and (11) and the following closed-loop system with sensor faults can be obtained.

$$\begin{aligned}\dot{x}(t) = (A_0 + \Delta A_0(t) + (B + \Delta B(t))KM_j(C + \Delta C(t)))x(t) + \\ \sum_{i=1}^{q}(A_i + \Delta A_i(t) + (B + \Delta B(t))F_i M_j(C + \Delta C(t)))x(t - h_i)\end{aligned} \tag{12}$$

Theorem 2. Suppose that the uncertain parameters of linear uncertain system with multiple time delays (4) satisfy

$$\| \Delta A_i \| \le a_i (i = 0,1,2,...,q), \| \Delta B \| \le b, \| \Delta C \| \le c$$

where $\| \bullet \|$ denotes the norm. $a_i (i = 0,1,2,...,q), b$ and c are certain fluctuant norm limit of model matrices.

If we use output feedback controller with multiple time delays (7) and the following condition can be retained,

$$\min_{M_i \in M} \lambda_m (S^j) > 0 \tag{13}$$

Linear uncertain closed-loop system with multiple time delays (12) possessing fault-tolerant for sensor failure $M_i \in M$ is given.

$$u(t) = Ky(t) + \sum_{i=1}^{q} F_i y(t - h_i) = -R^{-1}B^T Py(t) + \sum_{i=1}^{q} F_i y(t - h_i)$$

where P is positive definite solution of the following algebraic Riccati equation.

$$\begin{cases} A_0{}^T P + PA_0 - PBR^{-1}B^T PC - C^T PBR^{-1}B^T P + 2\alpha P + Q = 0 \\ \alpha \ge 0, Q \ge 0, R \ge 0 \end{cases}$$

where $\lambda_m (S^j)$ denotes the minimal eigenvalue of the following real symmetrical matrix.

$$\begin{aligned} S^j = & -(BKM_j C)^T P - PBKM_j C + Q + 2(\alpha - a_0)P + \\ & C^T PBK - bcM_j{}^T K^T P - bC^T M_j{}^T K^T P - cM_j{}^T K^T B^T P \\ & + PBKC - bcPKM_j - \sum_{i=1}^{q} \gamma_i{}^{-1} I_n - bPKM_j C - cPBKM_j \\ & - \sum_{i=1}^{q} \gamma_i D_j{}^T PE_j E_j{}^T PD_j \end{aligned} \tag{14}$$

$$K = -R^{-1}B^T P$$

where $D_j = A_0 + a_0 I_n + (B + bI_{n \times m})KM_j(C + cI_n)$.

$$E_j = A_i + a_i I_n + (B + bI_{n \times m})F_i M_j(C + cI_n)$$

Proof Define a Lyapunov function as

$$V(x,h) = x^T(t)Px(t) + \sum_{i=1}^{q} \gamma_i^{-1} \int_{t-h_i}^{t} x^T(\tau)x(\tau)d\tau$$

where P is positive definite solution of the algebraic Riccati equation(8), $\gamma > 0$. We get the differential coefficient of $V(x,t)$ against t along state trajectory ascertained by state function (12):

$$\dot{V}(x,h) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + \sum_{i-1}^{q}\gamma_i^{-1}x^T(t)x(t) - \sum_{i=1}^{q}\gamma_i^{-1}x^T(t-h)x(t-h) = [(A_0 + \Delta A_0(t) + (B + \Delta B(t))KM_j(C + \Delta C(t)))x(t) + \sum_{i=1}^{q}(A_i + \Delta A_i(t)) + (B + \Delta B(t))F_iM_j(C + \Delta C(t)))x(t-h)]^T Px(t) + x^T(t)P[(A_0 + \Delta A_0(t) + (B + \Delta B(t))KM_j(C + \Delta C(t)))x(t) + \sum_{i=1}^{q}(A_i + \Delta A_i(t)) + (B + \Delta B(t))F_iM_j(C + \Delta C(t)))x(t-h)] + \sum_{i=1}^{q}\gamma_i^{-1}x^T(t)x(t) - \sum_{i=1}^{q}\gamma_i^{-1}x^T(t-h)x(t-h) \le x^T(t)\{[A_0 + \Delta A_0(t) + (B + \Delta B(t))K M_j(C + \Delta C(t))]^T P + P[A_0 + \Delta A_0(t) + (B + \Delta B(t))KM_j(C + \Delta C(t))] + \sum_{I=1}^{q}\gamma_i^{-1}I_n + \sum_{i=1}^{q}\gamma_i[A_0 + \Delta A_0(t) + (B + \Delta B(t))KM_j(C + \Delta C(t))]^T P[A_i + \Delta A_i(t) + (B + \Delta B(t))F_iM_j(C + \Delta C(t))][A_i + \Delta A_i(t) + (B + \Delta B(t))F_iM_j(C + \Delta C(t))]^T P[A_0 + \Delta A_0(t) + (B + \Delta B(t))KM_j(C + \Delta C(t))]\}x(t) \le x^T(t)\{[A_0 + a_0I_n + (B + bI_{n\times m})KM_j(C + cI_n)]^T P + P[A_0 + a_0I_n + (B + bI_{n\times m})KM_j(C + cI_n)] + \sum_{i=1}^{q}\gamma_i^{-1}I_n + \sum_{i=1}^{q}\gamma_i [A_0 + a_0I_n + (B + bI_{n\times m})KM_j(C + cI_n)]^T P[A_i + a_iI_n + (B + bI_{n\times m})F_iM_j(C + cI_n)][A_i + a_iI_n + (B + bI_{n\times m})F_iM_j(C + cI_n)]^T P[A_0 + a_0I_n + (B + bI_{n\times m})KM_j(C + cI_n)]\}x(t) = -x^T(t)[-(BKM_jC)^T P - PBKM_jC + Q + 2(\alpha - a_0)P + C^T PBK - bcM_j^T K^T P - bC^T M_j^T K^T P - cM_j^T K^T B^T P + PBKC - b cPKM_j - \sum_{i=1}^{q}\gamma_i^{-1}I_n - bPKM_jC - cPBKM_j - \sum_{i=1}^{q}\gamma_i D_j^T PE_jE_j^T PD_j]x(t) = -x^T(t)T^j x(t) \le -\lambda_m(T^j) x^T(t)x(t)$$

As we all know, when condition (13) is held, $\dot{V}(x,h)$ will be smaller than zero. Based on the Lyapunov theorem, the closed-loop system (12) stability is retained. That is to say, output feedback controller with multiple time delays (7) has fault-tolerant ability for sensor failure $M_i \in M$.

3.3 Design Step

Based on above two theorems, fault-tolerant controller which has robust for actuator and sensor faults for a type of linear uncertain systems with multiple time delays can be designed according on the following steps:.

step1. Make certainly fluctuant norm limit of model matrices based on learning to transcendental knowledge of model uncertainty, for example $a_i(i = 0,1,2,...,q), b, c$;

Step2. Choice properly expected stabilization margin parameter α and weight parameters Q, R of Riccati equation (8)

Step3. Solve the Riccati equation and construct corresponding output feedback controller with multiple time delays;

Step4. Verify whether condition (6) and (13) is held. When condition (6) and (13) is hold, the corresponding output feedback controller with multiple time delays has ability of fault-tolerant against actuator faults and sensor faults. Otherwise, modify properly α, Q and R and resolve Riccati equation till condition (6) and (13) is held.

Ⅳ. Simylation Example

To illustrate the fault-tolerant control, a simple linear uncertain system with multiple time delays is simulated, and its dynamics are

$$A_0 = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}, A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Delta A_0(t) = \begin{bmatrix} 0.1\sin(t) & 0 \\ 0 & 0.1\cos(t) \end{bmatrix}, \Delta A_1(t) = \Delta A_2(t) = \begin{bmatrix} 0.1\sin(t) & 0 \\ 0 & 0.1\cos(t) \end{bmatrix}$$
$$\Delta B(t) = \begin{bmatrix} 0 & 0.1\sin(t) \\ 0.1\cos(t) & 0 \end{bmatrix}, \Delta C(t) = \begin{bmatrix} 0 & 0.1\sin(t) \\ 0.1\cos(t) & 0 \end{bmatrix}$$

We can know that $(A_i, B)(i = 1,2)$ is controllable.

Consider the following actuator fault models:

$$L = \{L_0, L_1, L_2\} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$$

Consider the following sensor fault models:

$$M = \{M_0, M_1, M_2\} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$$

Choice α=0, $Q = \begin{bmatrix} 10 & 3 \\ 2 & 8 \end{bmatrix}, R = I_2, \gamma_1 = \gamma_2 = 1$.

Based on (6) and (13), we can get a feasible $F_i (i = 1,2)$ as follow:

$$F_1 = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}, F_2 = \begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}$$

Solve the Riccati function (8) and get a positive definite matrix P

$$P = \begin{bmatrix} 1.18 & 0.24 \\ 0.24 & 1.05 \end{bmatrix}$$

The corresponding output feedback controller with multiple time delays is displayed as follow：

$$u(t) = -\begin{bmatrix} 1.18 & 0.24 \\ 0.24 & 1.05 \end{bmatrix} y(t) + \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} y(t-h_1) + \begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix} y(t-h_2)$$

Table 1 displays eigenvalues of T^i when actuators go wrong under the controller (7).

Table 1. Relation between T^i and actuator s faults under the controller (7)

Fault models	L_0	L_1	L_2
Eigenvalues of T^i	11.86 6.29	6.87 7.49	10.66 4.44

According to theorem1, the closed-loop system stability is retained.

Table 2 displays eigenvalues of S^i when sensors go wrong under the controller (7).

Table 2. Relation between S^i and sensor faults under the controller (7)

Fault models	M_0	M_1	M_2
Eigenvalues of S^i	11.86 6.29	6.63 7.21	10.83 4.79

According to theorem1, the closed-loop system stability is retained.

Initial condition： $x(0) = [4 \quad 2]^T, \varphi(t_0) = 0, t_0 \in [-2,0), a_0 = a_1 = a_2 = b = c = 0.1$

Time-delay： $h_1 = 1s, h_2 = 2s$.

In order to compare with the controller that is put forward by author, output feedback controller without considering the time delay can be gotten in the same condition.

$$u(t) = -R^{-1}B^T Py(t) = -\begin{bmatrix} 1.18 & 0.24 \\ 0.24 & 1.05 \end{bmatrix} y(t) \tag{15}$$

Table 3 displays eigenvalues of T^i when actuators go wrong under the controller (15) .

Table 3. Relation between T^i and actuator s faults under the controller (15)

Fault models	L_0	L_1	L_2
Eigenvalues of T^i	4.63＋1.43i	1.31＋4.18i	4.06
	4.63－1.43i	1.31－4.18i	
	4..32		

Table 4 displays eigenvalues of S^i when sensors go wrong under the controller (15).

Table 4. Relation between S^i and sensor faults under the controller (15)

Fault models	M_0	M_1	M_2
Eigenvalues of S^i	4.63＋1.43i ＋1.03i	1.17＋4.22i	4.19
	4.63－1.43i －1.03i	1.17－4.22i	4.19

Plot zero input response comparative figures of the closed loop system when each actuator or sensor goes wrong under the controller (7) and the controller (15) by using Matlab6.5 [10].

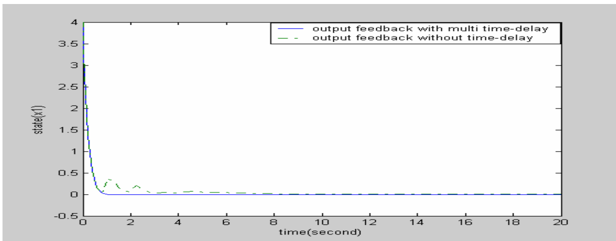

Fig. 1. Zero input response of state x1 in fault model L0

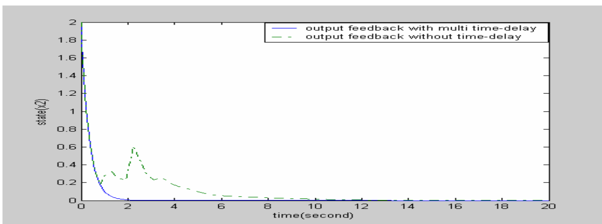

Fig. 2. Zero input response of state x2 in fault model L0

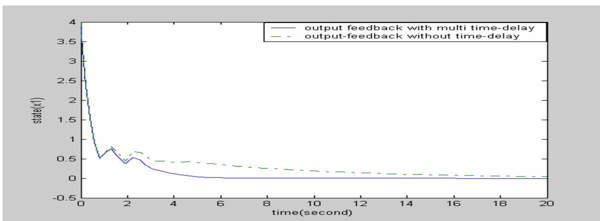

Fig. 3. Zero input response of state x1 in fault model L1

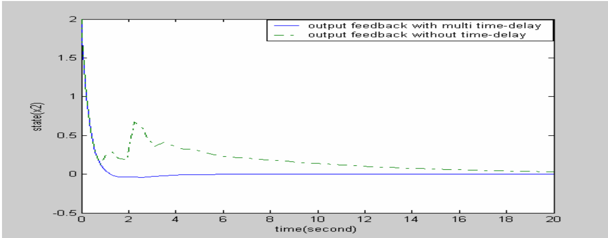

Fig. 4. Zero input response of state x2 in fault model L1

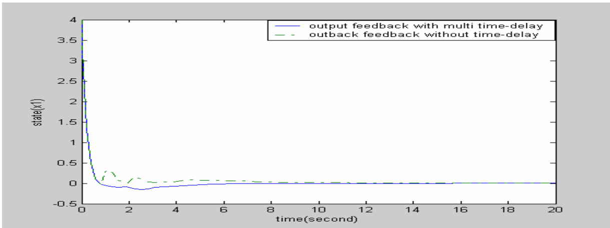

Fig. 5. Zero input response of state x1 in fault model L2

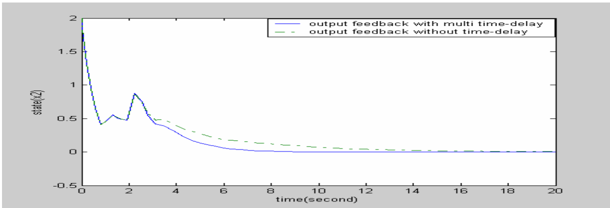

Fig. 6. Zero input response of state x2 in fault model L2

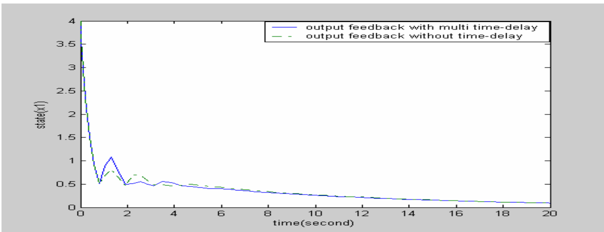

Fig. 7. Zero input response of state x1 in fault model M1

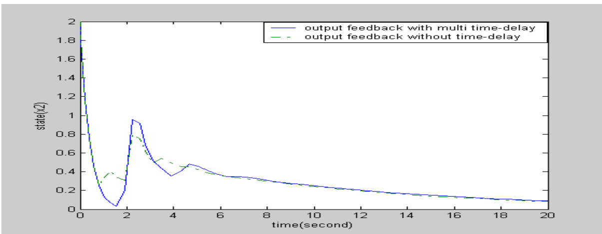

Fig. 8. Zero input response of state x2 in fault model M1

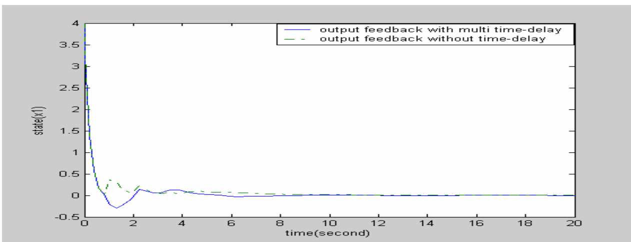

Fig. 9. Zero input response of state x1 in fault model M2

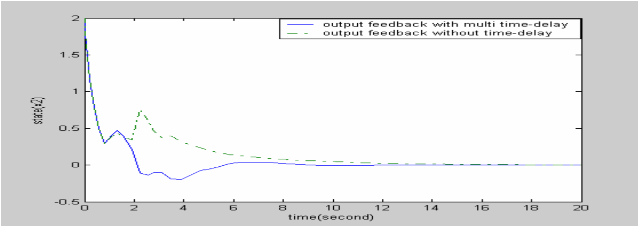

Fig. 10. Zero input response of state x2 in fault model M2

From Fig 1 to Fig 10, we can find that the two closed loop control systems are stabilized when actuator faults L1,L2,L3 and sensor faults M1,M2,M3 happen. However, the transition process of the feedback controller with multiple time delays outperforms clearly that of the output feedback controller without time delay. After we choice different time delay such as 1s,3s,4s and so on to emulate, we can find that the bigger the time delay is, the longer the stabilization time is. But the closed loop control system stability is always retained when actuators and sensors fail.

Ⅴ. Conclusion

The design of robust fault-tolerant controller for a type of linear uncertain systems with multiple time delays is studied. Based on Riccati's equation and Lyapunov's theorem, a type of output feedback controller with multiple time delays is put forward. Sufficient conditions for linear uncertain systems with multiple time delays which possess integrity for actuator and sensor failures are given. At last, simulation is given to demonstrate the effectiveness of the proposed approach and better stability criterion can be obtained compared to the case of output feedback without time delay.

Ⅵ. Acknowlegement

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