Formation Compound Control for Nonholonomic Mobile Robots under Motion Constraints

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Abstract

The formation keeping characteristics for nonholonomic autonomous mobile robots are investigated in this paper. A novel approach of compound control is presented, which deals with the formation task based on two different algorithms: a flexible feedback linearization control algorithm and an optimal approximate target algorithm. And the algorithms are selected according to robot's expected location and motion constraint domain. Also the formation stability is discussed to complete a steady formation. The proposed method can not only reduce the limitation of initial errors than traditional feedback linearization, rectify the formation shape distortion due to collision, but also resolve the unstable controlling problem while robots are turning rapidly. The experimental results have demonstrated the method's effectiveness.

Keyword: nonholonomic robot; formation keeping; feedback linearization

I. Introduction

Cooperation of multiple mobile robots has drawn an extensive research attention recently. It may have applications in transportation of large objects, geographical exploration, target searching, etc. Yet formation control, as the basic cooperative capability for these applications, includes formation building, formation keeping and formation switching. The formation keeping is the most key technology in the nonholonomic autonomous mobile robotics research community and the existing approaches to it fall into three categories. The behavioral method[1,2] is a distributed implementation and has explicit information feedback between neighbors. However, it is difficult to analyze mathematically. As an alternative to behavioral method, the virtual structure approach[3,4] has high precision, but it is unreliable due to a centralized implementation. Leader-follower approach[5,6,7] is very efficient and can be implemented easily though it needs some improvement.

In the 1980s, feedback linearization method, presented by R. Su firstly, was used to solve the nonlinear problems by coordinate transformation and state feedback. In 1998, J. P. Ostrowski considered the rigid formation control with small initial error in leader-follower pair by feedback linearization, which can stabilize the relative distance and orientation of the follower. However, in many practical applications, the uniform convergence rate of the method is slow due to large initial error and stochastic interferences, such as collision. Also the follower's velocity restriction is not considered explicitly.

Enlightened by the above approaches, a flexible and robust framework needs to be constructed to achieve the formation characteristics. In this paper, modeling of motion constraint is proposed to complete the motion domain partition of follower, whose desired position is estimated according to the formation requirements. In this way a compound control approach is presented under the surjective condition between desired position and motion constraint domain. The approach is composed of a new feedback linearization control algorithm, which manages followers to follow their leaders with bidirectional movement, and an optimal approximate target algorithm, which deals with unsteady movement due to follower's mechanical restriction. With the compound control approach discussed above, a non-rigid formation can be implemented steadily.

II. Modeling of Motion Constraint

Consider the planar motion of nonholonomic autonomous robots, The n^{th} robot is R_n $(n=1,2\cdots)$, whose body coordinate system C_n is embedded at the center of the drive axis, and C_n rotates with the robot R_n. World coordinate system C_0 is a fixed reference frame at the formation surroundings, whose kinematics are determined by

$$\begin{bmatrix} \dot{x}_n(t) \\ \dot{y}_n(t) \\ \dot{\theta}_n(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_n(t) \\ \omega_n(t) \end{bmatrix} \tag{1}$$

where $x_n(t)$, $y_n(t)$ are the coordinate values of R_n and $\theta_n(t)$ is the orientation in C_0, $(x_n, y_n, \theta_n) \in SE(2)$. $v_n(t)$ and $\omega_n(t)$ are the linear and angular velocities, respectively.

In practical applications of that R_j follows R_i, it is significant to know the exact accessible domain of R_j under the restriction of velocities. Then we defined the domain that are accessible for R_j in a control period as R_j's motion constraint domain.

The speed of left driving wheel and that of right driving wheel on robot R_j are v_l and v_r, respectively, given by

$$\begin{cases} v_l = v_j + a\omega_j / 2 \\ v_r = v_j - a\omega_j / 2 \end{cases} \tag{2}$$

where a is the distance between two driving wheels. The control input $u_j = [v_j \quad \omega_j]^T$ must satisfy the field D_j defined by $-V \le v_j + a\omega_j/2 \le V$ and $-V \le v_j - a\omega_j/2 \le V$, viz. $u_j \in D_j$. V is the maximum positive speed of wheels. R_j's motion constraint domain F_j can then be showed with symmetrical cardioid domains, see Fig. 1.

The motion constraint domain F_j of robot R_j can be modeled by its boundary curve c_f given by

$$\begin{cases} x_c = (V - a\omega_j \cdot sign(\omega_j)/2) \cdot \sin\omega_j T \cdot sign(v_j)/\omega_j \\ y_c = (V - a\omega_j \cdot sign(\omega_j)/2) \cdot (1-\cos\omega_j T) \cdot sign(v_j)/\omega_j \end{cases}. \tag{3}$$

Limiting form of the above model equation is $x_c = \pm VT$, $y_c = \theta_j = 0$ as $\omega_j \to 0$. Fig.1 depicts the domain boundaries, where E_{pi} denotes a positive motion constraint domain as $v_j > 0$, E_{pi} denotes a negative motion constraint domain as $v_j < 0$ oppositely, and E_o is an inaccessible domain for R_j in the next single control period. It can be seen that R_j can have a maximum orientation of $2VT/a$ radian and a distance of VT meters at most over a control period.

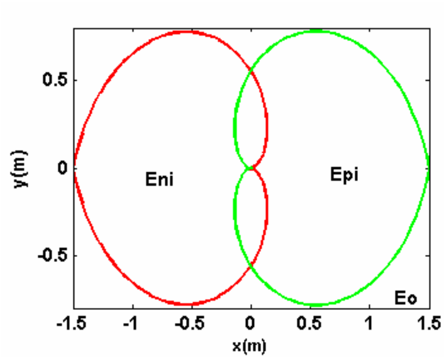


Fig. 1. Motion constraint domain

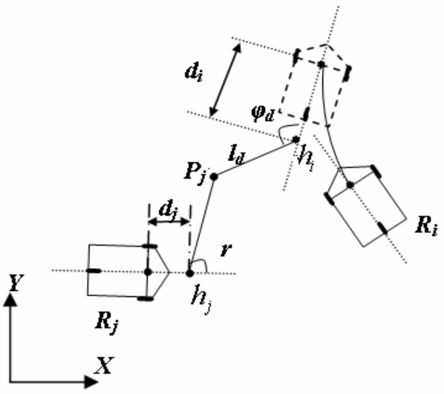


Fig. 2. Desired target estimation

III. Formation Compound Control Algorithm

A. *Following desired-target estimation*

In a simple leader-follower configuration (see Fig.2), robot R_j follows R_i with a desired distance l_d and desired relative bearing φ_d . Note that $d_n (n = i, j)$ is the separation between the axle midpoint of robot and the formation controlled point $h_n (n = i, j)$, which can be transferred to any point along the axes of robot according to the cooperation requirements, so that it enhances the flexibility of cooperation.

According to the relative positions and velocities of the robots, the next position can be predicted by the following estimation model

$$S_i(k+1/k) = f(u_i(k)) + g(d_i, l_d, \varphi_d) \tag{4}$$

$$P_j(k+1/k) = \mathrm{Ref}_{C_i\text{-}C_j}(S_i(k+1)) + h(r(k), d_j) \tag{5}$$

where S_i and P_j are points, which can be denoted by their coordinate values. $f(\cdot)$ is trajectory control function of leader R_i , $g(\cdot)$ is the transition function of the controlled point, $\mathrm{Ref}_{C_i\text{-}C_j}(\cdot)$ is the coordinate transform function from C_i to C_j . $h(\cdot)$ is the transition compensatory function of the controlled point. Since robot R_i is regulated by nonholonomic kinematics equation (1), desired target S_i in coordinate C_i is represented as follows

$$\begin{cases} x_{S_i}(k+1/k) = v_i(k)\cdot\sin(\omega_i(k)T)/\omega_i(k) - l_d\cdot\cos\xi(k) - d_i\cdot\cos(\omega_i(k)T) \\ y_{S_i}(k+1/k) = v_i(k)\cdot(1-\cos\omega_i(k)T)/\omega_i(k) - l_d\cdot\sin\xi(k) - d_i\cdot\sin(\omega_i(k)T) \\ \xi(k) = \omega_i(k)T + \varphi_d - \pi \end{cases}. \tag{6}$$

Point S_i transforms from coordinate system C_i to C_j, and then the coordinate values of the desired target P_j are given by

$$\begin{cases} x_{P_j}(k+1/k) = x_m(k) + x_{S_i}(k+1)\cdot\cos(\beta(k)) - y_{S_i}(k+1)\cdot\sin(\beta(k)) + \varepsilon_x \\ y_{P_j}(k+1/k) = y_m(k) + x_{S_i}(k+1)\cdot\sin(\beta(k)) + y_{S_i}(k+1)\cdot\cos(\beta(k)) + \varepsilon_y \end{cases} \tag{7}$$

where $(x_m(k), y_m(k), \beta(k))$ is the position of robot R_i in coordinate C_j. Transition compensatory term $\varepsilon_x = \pm \mathrm{d}_\mathrm{j}\cdot\cos(\gamma)$, $\varepsilon_y = \pm \mathrm{d}_\mathrm{j}\cdot\sin(\gamma)$, whose signs are positive or negative when R_j follows R_i on the left side or on the right side.

B. *Feedback linearization control algorithm (FBLC algorithm)*

Now consider a feedback linearization control algorithm for a single leader-follower pair. Nonlinear kinematics model of formation can be transformed to the linear system by state feedback laws as soon as its number of generalized coordinates equals to the dimension of system input vector.

Let the discrete-time dynamic model of the system be $z(k+1) = f(z(k), u(k))$, where $f(\cdot)$ is nonlinear and time invariant, system input $u(k) = [v(k), \omega(k)]^T \in D_j$, and $v_j \in [-V, V]$, so that follower R_j can be controlled in the kinematics system with bidirectional movement. The following direction is relevant to the angle γ, which deflects from the orientation of R_j to the desired target. The formation kinematics equations for R_j is given by

$$\delta(k+1) = \begin{cases} -V(|\gamma(k)| - \pi/2)n/\pi - v_j(k) + V & v_j(k) \geq 0 \\ -V(|\gamma(k)| - \pi/2)n/\pi - v_j(k) - V & v_j(k) < 0 \end{cases}, -\pi < \gamma \leq \pi, \tag{8}$$

$$\dot{l}_{ij} = v_j\cos(\theta_i - \theta_j + \varphi) - v_i\cos\varphi + d_j\delta\omega_j\sin(\theta_i - \theta_j + \varphi)/|\delta| - d_i\omega_i\sin\varphi, \tag{9}$$

$$\dot{\varphi} = v_i\sin\varphi/l_{ij} - \omega_i - v_j\sin(\theta_i - \theta_j + \varphi)/l_{ij} - d_i\omega_i\cos\varphi/l_{ij} + d_j\delta\omega_j\cos(\theta_i - \theta_j + \varphi)/(|\delta|\, l_{ij}) \tag{10}$$

where δ is following direction factor, $n \geq 4$. $\dot{l}_{ij} = k_1(l_d - l)$, $\dot{\varphi}_{ij} = k_2(\varphi_d - \varphi)$, k_1 and k_2 are controller gains. l is the actual distance from desired target P_j to h_j, φ is the relative bearing from the orientation of robot R_i to line $P_j h_j$. Note that discrete-time k is omitted in equations (9) and (10).

The constrained relationship of factor δ and angle γ is defined by the equation (8), so that the explicit equations of system input can be deduced from the equations (8-10) as follows

$$u_j(k+1) = A + Bu_i(k) \tag{11}$$

where A and B are matrix functions with variables l and φ, B is nonsingular matrix and the dimension of auxiliary control input u_i equals to that of u_j, so that the I/O response of the closed-loop system is linear.

According to the value of direction factor δ, robot R_j can adjust the following direction to its leader adaptively. Robot R_j follows R_i in a positive direction only if $\delta \geq 0$, which occurred in two kinds of states. One is that R_j is going ahead towards the desired target. The other is that current speed v_j is less than $V \cdot (sign(v_j) - \rho n/\pi)$ though R_j is deviating from the desired target, where $\rho = |r| - \pi/2 \in (0, \pi/n)$ is buffer bearing, which is helpful to the control stability. For other conditions, robot R_j follows R_i in an inverse direction.

C. *Optimal approximate target algorithm (OAT algorithm)*

A new approximate target is needed when robot R_j can't reach the desired target, which is not considered especially in traditional formation approaches. To resolve the problem, an optimal approximate target algorithm is presented.

Passing through the point $P_j\ (x_{P_j}, y_{P_j})$, perpendicular lines of tangential direction of boundary curve c_f can be written as

$$y = -\mathrm{ctan}(\omega_j T) \cdot (x - x_{p_j}) + y_{p_j}\,. \tag{12}$$

In case of that point $G(x_v, y_v)$ is the foot of the above perpendicular, also it is on the boundary curve c_f, then x_v, y_v must satisfy the equations as follows

$$\begin{cases} y_v = -\mathrm{ctan}(\omega_j T) \cdot (x_v - x_{p_j}) + y_{p_j} \\ x_v = (V - a\omega_j \cdot sign(\omega_j)/2) \cdot \sin \omega_j T \cdot sign(v_j)/\omega_j \\ y_v = (V - a\omega_j \cdot sign(\omega_j)/2) \cdot (1 - \cos(\omega_j T)) \cdot sign(v_j)/\omega_j \end{cases}. \tag{13}$$

Above equations can be simplified as follows

$$(\mathrm{V}/\omega_j - a \cdot sign(\omega_j)/2) \cdot sign(v) = \mathrm{ctan}(\omega_j T) \cdot x_{p_j} + y_{p_j}\,. \tag{14}$$

In addition, the sign functions of system inputs can be represented by those of coordinate values of point $P_j\ (x_{P_j}, y_{P_j})\sqrt{a^2 + b^2}$ as follows

$$\begin{cases} sign(v_j) = sign(x_{P_j}) \\ sign(\omega_j) = sign(x_{P_j}) \cdot sign(y_{P_j}) \end{cases}. \tag{15}$$

Substituting the equation (15) into the equation (16) and considering the constraint boundary of system input, the formation control law is given by

$$\begin{cases} v_j = V \cdot sign(x_{P_j}) - a\omega_j \cdot sign(y_{P_j})/2 \\ \mathrm{V} \cdot sign(x_{P_j})/\omega_j - a \cdot sign(y_{P_j})/2 = \mathrm{ctan}(\omega_j T) \cdot x_{P_j} + y_{P_j} \end{cases}. \tag{16}$$

In this way, the optimal approximate target is the solution which satisfies $\min(|GP_j|)$, and the corresponding v_j and ω_j are the valid system control inputs.

D. Implementation of formation compound control

Consider the point $J(x_e, y_e)$, which is on the boundary curve c_f along the direction to the desired target in C_j, and its coordinate values (x_e, y_e) can be calculated while $\omega_j = (2/T)\arctan(y_{P_j} / x_{P_j})$, so that an operation rule function can be obtained as follows

$$cost(x_{P_j}, y_{P_j}) = \| P_j O_j \| - \| JO_j \| \tag{17}$$

where O_j is the origin of coordinate system C_j, $\| AB \|$ represents a distance between A and B.

Feedback linearization control algorithm depicted in section 3.2 will be used to the formation control when $cost(\cdot) < 0$, which happens as desired target is in the constraint domain F_j, i.e. $P_j \in \mathrm{E}_{pi} \bigcup \mathrm{E}_{ni}$; Optimal approximate target algorithm depicted in section 3.3 will be used to the formation control when $cost(\cdot) > 0$, which happens as desired target is out of the constraint domain F_j, i.e. $P_j \in \mathrm{E}_o$; and ω_j satisfying $cost(\cdot) = 0$ will be the system input directly. Since the rules of the compound control are subjective, the formation system is a controlled system.

IV. Stability Analysis of Formation

For any system input $u_j = [v_j, \omega_j]^T \in D_j, \omega_j \neq 0$, the leader's input $u_i = [v_i, \omega_i]^T \in D_i$ must satisfy

$$|\omega_j| \leq \frac{V_j}{\sqrt{l_d^2 + (v_i / \omega_i)^2 + 2l_d \cdot (v_i / \omega_i) \cos(|\frac{3}{2}\pi - \varphi_d|)}}, \tag{18}$$

so that the formation characteristics can meet the requirement of accuracy.

For above definition, it can be analyzed with geometry approach. Assume that robot R_j follows R_i with a desired distance l_d and desired relative bearing φ_d. Given a time slice, R_i has an arc trajectory with radius r_i, so that the desired trajectory of follower R_j should be a homocentric arc with equal central angle.

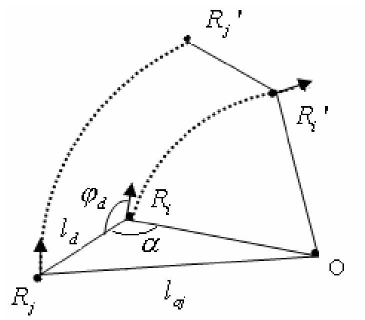


Fig.3. velocities constraint of stable formation

Suppose $\omega_i < 0$, then

$$v_j = |\omega_j| \cdot l_{oj} \tag{19}$$

where l_{oj} is the distance from R_j to the centre O (see Fig.3), which can be calculated from the triangle $\Delta R_i R_j O$ as follows

$$l_{oj} = \sqrt{l_d^{\,2} + r_i^{\,2} - 2l_d \cdot r_i \cos(|\frac{3}{2}\pi - \varphi_d|)} \tag{20}$$

where $r_i = |v_i / \omega_i|$. In addition, the value of l_{oj} can be transformed to (20) when $\omega_i > 0$, and $v_j \le V_j$, such that conditional inequality(18) is deduced. Note that above definition regards the robot as point robot. For non-point robots, it needs to be adjusted according to the difference of controlled points on the robot.

Conditional inequality (18) defined the steady requirement to specifically formation. If it isn't satisfied, u_i should be adjusted by a feedback coefficient $\mu(P_j, G)$, which is the function of desired target $P_j(x_{P_j}, y_{P_j})$ and optimal appropriated target $G(x_v, y_v)$ as follows

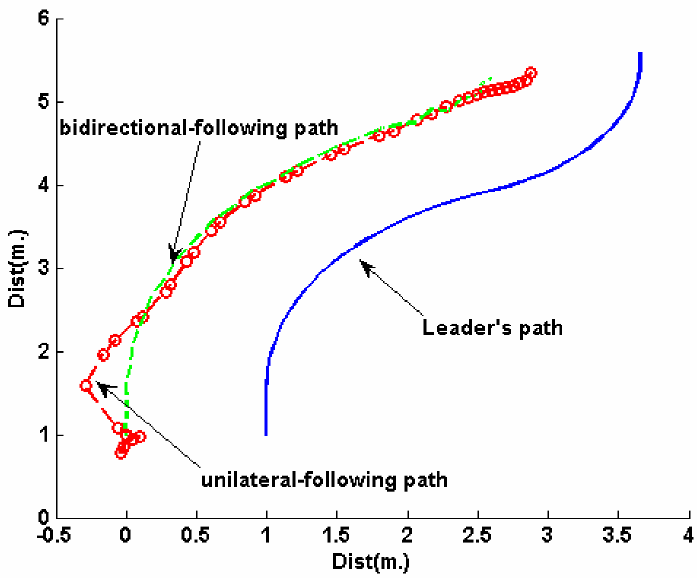
$$\mu(P_j, G) = \begin{cases} \eta \cdot \exp(-\sqrt{(x_{P_j} - x_v)^2 + (y_{P_j} - y_v)^2}) & P_j \in E_o \\ 1 & P_j \in E_{pi} \cap E_{ni} \end{cases} \tag{21}$$

where η is a user-selected proportional constant, so that the control input of leader is automatically adjusted to adapt to the formation states.

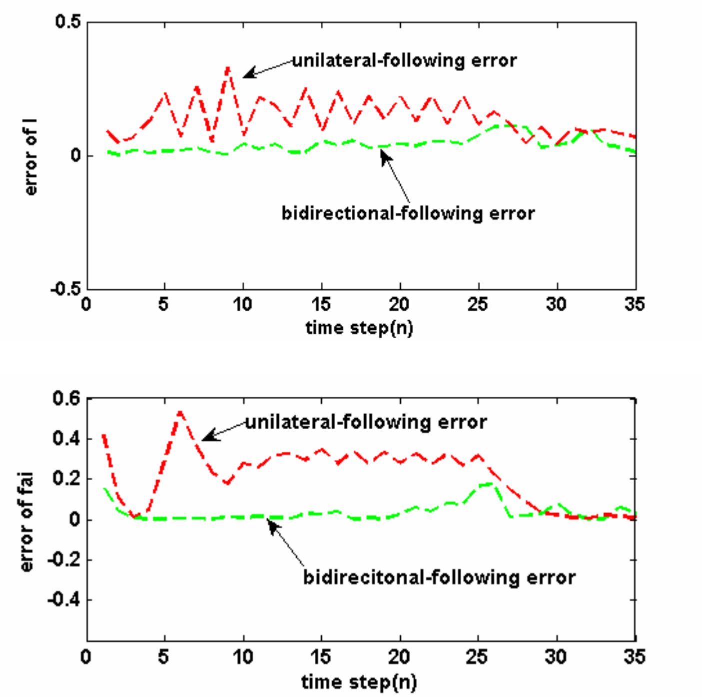
V. Experiments

The formation compound control algorithm was implemented in our experiments. Each robot travels at normal velocity of 0.1 m/sec and a maximum velocity of 1 m/sec, the distance between two driving wheels is 0.4 m, and control period is 1.5 sec.

Initial experiment is a line formation experiment with $l_d = 1$ and $\varphi_d = \pi/2$. The initial position of leader R_1 is $(1,1,\pi/2)$, and that of follower R_2 is $(0,1,3\pi/2)$.



(a) Comparison of formation path



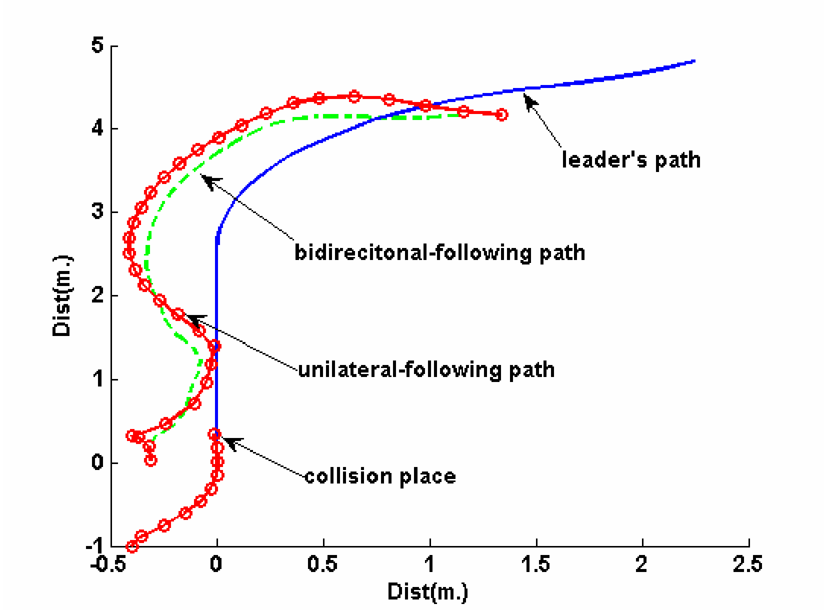
(b) Comparison of formation errors

Fig.4. Line formation with or without bidirectional control

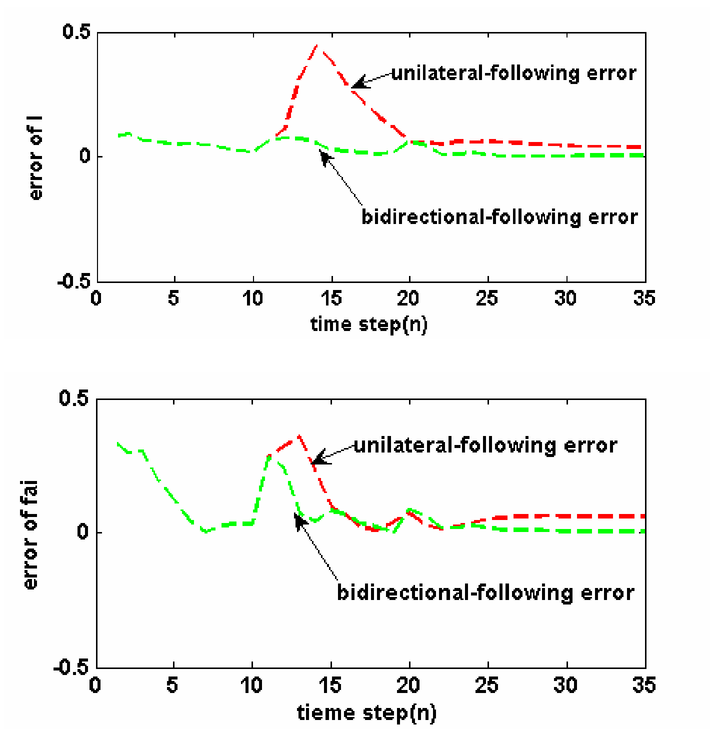
As can be seen in Fig.4, traditional unilateral following control has a large initial bearing error, which results in an imprecise trajectory contrary to bidirectional following control with feedback linearization. Fig.4 (b) contrasts the leader-follower separation(l) errors and bearing(fai=φ) errors. Results are that bidirectional following motion motivated by feedback linearization control algorithm can ensure the formation initial bearing error no more than $\pi/2$, which enhances the precision of formation greatly.

Additional experiment with an accidental collision in a column formation was also conducted. In this instance, formation parameters satisfy $l_d = 1, \varphi_d = \pi$. Note that follower R_2 is transferred from position $P_2(x_2, y_2, \theta_2)$ to $P_2'(x_2 - 0.3, y_2 - 0.3, \theta_2 - \pi)$ at time $t = 12$ sec.

Shown in Fig.5, while the bidirectional control method was developed to handle large initial error, it can accommodate the case where follower is collided. At the collision place, the sign of the direction factor δ which is calculated in equation (8) changes to minus instead of positive one, so that R_2 alters its following direction to recover the formation shape quickly. While unilateral following robot has to turn to original following direction, which results in a big disturbance of formation error.



(a) Comparison of formation path



(b) Comparison of formation errors

Fig.5. Column formation with or without bidirectional control

VI. Conclusion

In this paper, a compound control scheme for robot formation is proposed. The main contributions are a series of control and estimation algorithms. According to the nonholonomic characteristics of the mobile robots, motion constraint domain is defined. Especially, a feedback linearization control algorithm with bidirectional following characteristics and an optimal approximate target algorithm are presented. Compared with the traditional leader-follower formation methods, the compound control approach enhances the flexibility and robustness of formation control, and it is also applicable in the cooperative patrolling and environment exploring. Deciding the optimal formation shape for a given environment and implementing coordination tasks with more robots are also important directions for our future work.

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