An Improved Dempster-Shafer Algorithm for Resolving the Conflicting Evidences

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Abstract

Dempster-Shafer evidence theory provides a useful computational scheme for integrating uncertainty information from multiple sources in artificial intelligence systems. Therefore, it has been successfully applied in data fusion and pattern recognition. However, it also has some shortcomings. D-S evidence combination can not proceed if the evidence totally collides with each other. In addition, the combination result can not be kept according to the real condition when the evidence seriously conflicts. To solve this problem, this paper presents an improved D-S algorithm, which verifies and modifies the conflicting evidences. Experiments show that this method improves the reliability and rationality of the combination results. Although evidences highly conflict with one another, the combination result still satisfies the practical situation.

Keyword: conflicting evidences, D-S evidence theory, data fusion

I. Introduction

In pattern recognition, the information extracted from the sensors are often represented by a degree of belief resulting from imprecise and uncertain data. The multi-sensor data fusion is an interesting solution in order to gather more reliable information. The Dempster-Shafer evidence theory has been widely discussed and used recently, because it is a reasonable, convenient, and promising method to combine uncertain information from disparate sources with different levels of abstraction [1-3]. Unfortunately, although D-S theory is widely used in data fusion systems, there are some basic problems still not completely clarified.

The existing combination rules can be distinguished in two categories. The first type of combination operators imposes the hypothesis of reliability of all sources which have to be aggregated. These conjunctive operators have been introduced by Dempster and Smets[4]. This first rule carries out some problems. The first problem comes from the idempotence, that is to say that the combination of two dependent sources allows to reinforce the propositions that these sources sustain abusively. The second appears in case of conflict between the sources. In this case, the Dempster combination rule proceeds a step of normalization**.** This problem, known as sensitivity of the Dempster rule, has been presented by Zadeh. In [5], Zadeh presented a situation where the step of normalization used by Dempster's combination rule to results against intuitive.

The second category imposes that at least one of the information sources is reliable. The second kind includes the disjunctive operators that have been presented by Yager[6]. These operators consider that at least one of the sources concerned with the fusion is reliable, but without knowing which is reliable. In order to cure this problem, other combination operators have been proposed[6-8]. However, unfortunately none of them are complete.

To solve this problem, this paper presents a new method to verify and modify the conflicting evidences. In this method, additive strategy is adopted to make events to be descending firstly, then the q events which are most likely to recognize results are sought out. Next, verify and modify them using the method proposed in this paper. Finally, simulation experiments are conducted to demonstrate the rationality of this new improved algorithm when dealing with conflicting evidences.
The organization of this paper is as follows. Section 2 starts with a brief presentation of the D-S evidence theory. In section 3, we will discuss the shortcomings of evidence theory whiling dealing with conflicted evidences. An improved D-S algorithm will be introduced in section 4. Section 5 describes simulation experiments that demonstrate the proposed method in this paper. Finally, Section 6 draws the conclusions.

II. Basics of the D-S Evidence Theory

The evidence theory was first introduced by Dempster in the 1960s, and later developed by Shafer in 1976. Here, the main concepts of the D-S theory are briefly recalled and some basic notation is introduced.

Let U be a set of mutually exclusive and exhaustive hypotheses about some problem domain, which is referred to as the frame of discernment. The D-S theory started with the idea of using a number in the interval [0,1] to indicate the degree of evidence supporting a proposition.
Based on observing evidence E, the function $m(\bullet)$ provides the following basic probability assignments (BPAs) on U :

$$\begin{cases} m(\phi)=0 \\ \sum_{A\subset U} m(A)=1 \end{cases} \tag{1}$$

For any $A\subset U$, $m(A)$ represents the belief that one is willing to commit exactly to A, given a certain piece of evidence.
The subset A of frame U is called the focus element of m , if $m(A)>0$.
The belief measure and the plausibility measure of a proposition are, respectively, defined as follows:

$$BEL(A)=\sum_{B\subset A} m(B) \quad (\forall A\subset U) \tag{2}$$

$$PL(A)=1-BEL(\overline{A})=\sum_{B\cap A\neq\phi} m(B) \tag{3}$$

where $BEL(A)$ and $PL(A)$ represent the lower bound and upper bound of belief in A. Hence, interval $[BEL(A), PL(A)]$ is the range of belief in A .

If m_1 and m_2 are two BPAs induced from two independent evidence sources, and the condition $\sum_{\substack{i,j \\ A_i \cap B_j = \phi}} m_1(A_i) m_2(B_j) < 1$ is met, then the combined BPA can be calculated according to Dempster's rule of combination:

$$\begin{cases} m(C) = \dfrac{\sum_{\substack{i,j \\ A_i \cap B_j = C}} m_1(A_i) m_2(B_j)}{1 - \sum_{\substack{i,j \\ A_i \cap B_j = \phi}} m_1(A_i) m_2(B_j)} \\ m(\Phi) = 0 \end{cases} \tag{4}$$

The combination of m is possible if and only if there exist at least two subsets A_i and B_j with $A_i \cap B_j = \phi$ such that $m_1(A_i) \neq 0$ and $m_2(B_j) \neq 0$. m_1 and m_2 are then said to be combinable. The combination rule is commutative and associative, so it can be easily extended to combine several belief functions by repeating the rule for new belief functions.

Let $k = \sum_{\substack{i,j \\ A_i \cap B_j = \phi}} m_1(A_i) m_2(B_j)$, which embodies the degree of conflict between evidence sources. The coefficient $\dfrac{1}{1-k}$ is called normalization factor and is used to avoid non zero mass from being assigned to the empty set after combination.

III. Shortcomings of Dempster's Rule of Combination with Conflicted Evidences

Let k embody the degree of conflict between evidence sources with the constraint $0 \le k \le 1$. Larger k is, highly conflicts existing evidences. If k is close to 1, unreasonable fusion result is more likely to reach. If k equals to 1, that is to say, evidences are completely conflicting, Dempster's rule of combination of evidence can not be applied directly. The instances are as follows:

1) When evidences are highly conflicting, fusion result by using eq.(4) will be improper.
Assume that there are two BPAFs m_1 and m_2 on the frame of discernment $U = \{A, B, C\}$, and $m_1(A) = 0.99$, $m_1(B) = 0.01$, $m_1(C) = 0.0$, $m_2(A) = 0.0$, $m_2(B) = 0.01$, $m_2(C) = 0.99$.

According to combination eq.(4), the integrated BPA function m becomes as follows. $m(A) = m(C) = 0$, $m(B) = 1$.

From the fusion results, we can see that the two evidences with infinitesimal supporting of B result in an almost affirmative supporting. Apparently, that is very strange in the light of human common sense. Aimed at this situation, Yager had done some research. The method proposed by Yager[6] follows this principle. While considering that at least one of the sources concerned with the fusion is reliable, but without knowing which is reliable, the conflict is then distributed on the set U. The conflicting mass is placed therefore on U. This method has the effect of separating the totality of the conflicting mass, and more to intervene in the discernment of the hypotheses. Unfortunately, it is not associative. Therefore, it is necessary to define an order of fusion of the sources therefore.

2) Tiny changes of basic probability assignment function can bring sharp changes of fusion result. Assume a little modification on m_1 mentioned above, and m_2 is invariable. $m_1(A)$=0.98, $m_1(B)$=0.01, $m_1(C)$=0.01.

Then, $m(B)=0.01$ .Compare with the result $m(B)=1$ from 1), combination results are almost contrary. It reflects that the combination rule is sensitive to BPA function.
3) If there are disaccords between one piece of evidence and several other evidences, irrational result will be induced.

For example, $m_1(A)=0.98$, $m_1(B)=0.01$, $m_1(C)=0.01$, $m_2(A)=0.0$, $m_2(B)=0.01$, $m_2(C)=0.99$.
If $m_1=m_3=m_4=\cdots=m_n$, then the combination result is $m(A)=m(C)=0$, $m(B)=1$. Obviously, it is unreasonable. If the combination rule proposed by Yager is adopted, then $m(A)=0$. In fact, that one or few sensors in multi-sensor system go wrong is possible, even frequent. So, Dempster's or Yager's rule of combination of evidence are not effectual. So, it is necessary to study the problem of combination of conflicting evidence future.

IV. Improved Fusion Method for Conflicting Evidences

The improved fusion method for conflicting evidences is described as follows.
In practical circumstances, conflicting evidences often appear because of hostile interference or malfunction of exceptional sensor. This is not our hope. So, modifications to the evidence must be made. Assume that $m_i(\cdot)$, $i=1,2\cdots m$ are BPA function. Events are noted as A_j, $j=1,2\cdots,n$. The basic probability assignment of the i^{th} evidence on the j^{th} event is called basic element, which is marked as m_{ij}.

Step 1. Verify the conflicting evidences.
In order to grantee the universality of the method, verifying the conflicting evidences is necessary.
When one basic element supports an event on the small side, and at the same time other basic elements on this event are all very big, the lesser basic element is considered to be exceptional. That is to say, it is conflicting. Additive strategy is adopted to find the biggest mass of event, and then judge whether it is conflicting or not.

In the same way, if a basic element on an event is bigger and other basic elements on this event are all very small, the bigger basic element is thrown out of the way. But we have enough reason to consider that it is not the recognition conclusion. So, this situation can be neglected.
Adopt additive strategy and let events be descending sort. Then, find out the former q events B_k.

Let

$$S_j = \sum_{i=1}^{m} m_{ij} \tag{5}$$

Let S_j be descending sort. The former q events are $\{B_k, k = 1,2\cdots,q\}$. Verifying the conflicting evidences will be conducted in set $\{B_k\}$. Considering the impact of proportion, basic share P_j that describes the impact of basic element on combination will be calculated as:

$$P_j = \frac{1}{m}\sum_{i=1}^{m} m_{ij}\ , \quad j = 1,2\cdots,q \tag{6}$$

Calculating the difference Q_{ij} between P_j and basic element m_{ij},

$$Q_{ij} = P_j - m_{ij} \tag{7}$$

$$S_{ij} = \frac{P_j}{Q_{ij}} \tag{8}$$

For a given threshold α $(\alpha > 1)$, if $1 < S_{ij} < \alpha$, then m_{ij} is judged as conflicting evidence, marked as m_{ik}. If $S_{ij} >= \alpha$ or $S_{ij} < 0$, then m_{ij} is judged as normal evidence.

If $1 < S_{ij} < \alpha$, then $0 < m_{ij} < \frac{\alpha - 1}{\alpha} p_j$. If $S_{ij} >= \alpha$ or $S_{ij} < 0$, then $m_{ij} \geq \frac{\alpha - 1}{\alpha} p_j$.

In this way, judgement condition that m_{ij} is conflicting evidence can be described as:

$$0 < m_{ij} < \frac{\alpha - 1}{\alpha} p_j \tag{9}$$

Judgement condition that m_{ij} is normal evidence can be described as:

$$m_{ij} \geq \frac{\alpha - 1}{\alpha} p_j \tag{10}$$

Step 2. Modify the conflicting evidences m_{ik}.

$$m_{ik} = m_{ik} + \frac{1}{l} \tag{11}$$

In eq.(11), $\frac{1}{l}$ is called modified pace. l is related to the number of event n. From the point of practice, modified pace is no more than $\frac{1}{n}$. In a general way, $l = 2n$. Then, normalization must be done to m_{ik} in order to satisfy eq. (1).

$$m'_{ij} = \frac{m_{ij}}{\sum_j m_{ij}} \tag{12}$$

Return to step 1, verify and modify conflicting evidences once again till it satisfies $m_{ij} \geq \frac{\alpha - 1}{\alpha} p_j$.

After verifying and modifying conflicting evidences, eq.(4) can continue to be applied to combine evidences.

V. Experiments and Analysis

Assume that there are three BPA functions m_1, m_2 and m_3 on the frame of discernment $U = \{A, B, C\}$. If the Dempster's rule of combination of evidence is directly employed, combination result can be seen in table 1.

Table 1. Fusion results using Dempster's rule of combination of evidence directly

	A	B	C
$m_1(\bullet)$	0.9	0.09	0.01
$m_2(\bullet)$	0.01	0.4	0.59
$m_3(\bullet)$	0.7	0.29	0.01
$m_1 \oplus m_2 \oplus m_3$	0.1768	0.7073	0.1159

From data above mentioned, we can see that B is the decision-making result. Obviously, the result is against intuitive.

Next, use the improved algorithm to verify and modify the conflicting evidence. Let the threshold α be 2. First, check up the evidences from each source. Then, modify the conflicted evidence. Finally, use Dempster's rule of combination of evidence to deal with the modified evidences. The fusion results are shown in table 2.

Table 2. Fusion results after verifying and modifying the conflicted evidences

	A	B	C
$m_1(\bullet)$	0.56	0.236	0.204
$m_2(\bullet)$	0.204	0.36	0.436
$m_3(\bullet)$	0.7	0.29	0.01
$m_1 \oplus m_2 \oplus m_3$	0.758	0.234	0.008

Apparently the final decision-making result is A, which accords with intuition. From the experiments above, we can see that the fusion method proposed in this paper can solve conflicted evidence well and conquer the shortage of Dempster's rule of combination of evidence when dealing with conflicted evidences.

To test validity of fusion method when dealing conflicted evidences further, in the example below, the D-S fusion method is applied to identify the type of hostile battleplane interfered with multisensor system.

Assume that a hostile battleplane with type A has a constant velocity, which has relative velocity of 1 km/s with our plane. The initial range of the two planes is one hundred kilometers apart.
There are three sensors on our plane to offer type information about hostile plane. Sensors offer data once per second. And the accuracy will be reduced with increasing distance between two planes. When the distance is 10 kilometers, the accuracy of data offered is 80%. When the distance is 100 kilometers, the accuracy of data offered is 40%. When the two planes are from 20 kilometers to 30 kilometers apart, a sensor on our plane is interfered. In this interval, the accuracy of data offered is 10%. In the simulation below, the threshold α is set to be 2.

Figure 1 shows the BPA function of proposition of hostile battleplane with type A. In figure 1, the real line shows the result of using the traditional D-S combination rule directly. The broken line shows the result of utilizing the modified algorithm on conflicted evidence.

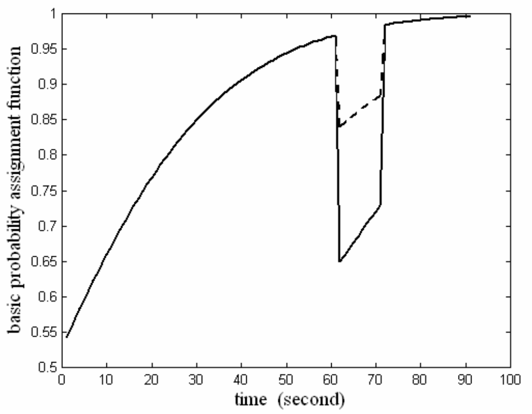

Fig. 1. BPA function of proposition of hostile battleplane with type A

From figure 1, we can see that when the evidences conflict highly, the traditional D-S fusion method results in descending soon of belief degree of correct proposition. That will produce false assignment. Moreover, if the method proposed in this paper is adopted, reliability of correct proposition descends little.

Thus it can be seen the fusion algorithm proposed in this paper can deal with highly conflicted evidences and get rid of the influence of interferer. The decision-making result accords with the fact.

VI. Conclusions

In this paper, we introduced the problem of the sensibility in case of conflict with Demspter's combination rule. An improved D-S fusion algorithm is proposed aiming at conflicting evidences. Using this algorithm, we can solve a controversial problem in D-S theory about how to combine conflicting evidences. The proposed method can deal with the highly conflicted evidences. And at the same time, the consistent evidence can also be combined effectively.

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