Control of A Multivariable High Purity Distillation Column Based on Closed-loop Gain Shaping Algorithm

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Abstract

Control of a high purity distillation column is a benchmark problem that demands strong robustness and is difficult to control in chemical industry because the plant of a high purity distillation column is ill-conditioned. Closed-loop gain shaping algorithm for MIMO system was presented, that constructed a robust controller by the four engineering parameters of the largest singular value, bandwidth frequency, peak value and slope of high frequency asymptote according to the desired shape of sensitivity function and closed-loop frequency spectrum. It was applied to a high purity distillation column. Simulation results show that it has good control effect and robust stability. The algorithm given in the paper has the advantages of simple design procedure and clear physical meaning.
Keyword: Closed-loop gain shaping High purity distillation column Robust control.

I. Introduction

Control of a high purity distillation column is a benchmark problem that demands strong robustness and is difficult to control in chemical industry because the plant of a high purity distillation column is a MIMO system with two inputs and two outputs, furthermore the condition number of the system is very large, i.e. the control system is ill-conditioned. The common robust algorithms such as H_∞ mixed sensitivity algorithm, loop shaping algorithm and μ analysis algorithm and so on are complex and demand good mathematic basis to learn, the solving of controller requests specific software and needs some work experience, so the application of these algorithms is difficult for an actual engineer[1,2]. A discrete-time multivariable globally linearized control (GLC) algorithm was given in reference [3] to control a constrained multivariable distillation column, through extensive numerical simulations the control law showed a high quality performance for set point tracking and disturbance rejection in presence of parametric uncertainty. A quasi-ARMAX modeling and control of a multi-variable ill-conditioned and nonlinear distillation column was presented in reference [4]. Reference [5] presented an application of fuzzy model-based methods to the analysis and control design of a high-purity binary distillation column. Reference [6] gave an inferential control methodology for an industrial multicomponent distillation column, that utilized an ANN estimator for a model predictive controller (MPC).

The mixed sensitivity problem of H$_\infty$ robust control theory is a kind of closed-loop gain shaping algorithm[7], shaping the compensatory sensitive matrix $T = GK(I+GK)^{-1}$ (which is also the transfer matrix of the closed-loop system, *G*, *K* are the transfer matrix of the controlled plant and the controller respectively) and the sensitive matrix $S = (I+GK)^{-1}$ through the weighting matrix W_1, W_2, making the system to satisfy the index of robust performance and robust stability. The algorithm solves the controller through norm optimizing of the closed-loop transfer matrix ($[W_1S \quad W_2T]^T$). It processes the singular value of the matrix in its deductive method instead of the transfer matrix itself, so it has to use a series of advanced mathematic tools. The solving process to acquire a robust controller is complex. This procedure is rigorous only in the theoretical sense, since the final solution to a great extent depends on the weighting matrix W_1, W_2 in shaping the sensitive matrix *S* and compensatory sensitive matrix *T* to satisfy the robust performance and robust stability of the system. Unfortunately the selection of the weighting matrix are almost arbitrary. The closed-loop system parameters with engineering sense such as bandwidth frequency, high frequency asymptote slope, the largest singular value and the peak value of the closed-loop frequency spectrum have no clear relationship with weighting function matrix, thus the design of robust controller using H$_\infty$ theory requires rich experience of weighting function matrix selecting, and brings some difficulty to the early learner [2].

Loop Shaping algorithm of H$_\infty$ control theory is a kind of open-loop gain shaping method[8]. Its key point lies in finding a controller *K* to make the gains $\underline{\sigma}(L)$ and $\bar{\sigma}(L)$ of the open-loop transfer function matrix *L*=*GK* satisfying robust performance in low frequency zone and robust stability in high frequency zone, i.e. high gain in low frequency zone and low gain in high frequency zone. Loop shaping algorithm implements the closed-loop performance of the system through selecting weighting functions to shape the open-loop frequency characteristic curve, and obtaining an acceptable performance/robustness trade-off. Its advantage is that the designer knows clearly how to change the shape of *L* to acquire a satisfactory controller *K* with fine robust performance and robust stability; however, it does not consider directly the closed-loop transfer functions, such as *S* and *T*, which determine the final response of the system.

Closed-loop gain shaping algorithm applies the true essence of H$_\infty$ mixed sensitivity algorithm to construct directly the compensatory sensitive function *T*. The construction uses four parameters with engineering sense i.e. bandwidth frequency, high frequency asymptote slope, the largest singular value and the peak value of the closed-loop frequency spectrum. The algorithm constructs indirectly the sensitive function *S* because of the correlativity between *S* and *T* (*T*=I−*S*), then the controller *K* is reversely deduced out[7,9]. It can be considered that the mixed sensitivity algorithm is the result of direct thought, and loop shaping algorithm is the result of divergent thought, while closed-loop gain shaping algorithm is the result of converse thought. The core of closed-loop gain shaping algorithm is designing the controller using directly the expression formula of the constructed closed-loop transfer function matrix of the system, while all the four parameters for constructing *T* have actual engineering sense, the robust performance and robust stability of the system are guaranteed because of the good shape of *T* and *S*. The advantage of the closed-loop gain shaping algorithm is that the physical meanings supporting the idea are clear and the solving procedure is relatively simple.

II. Closed-loop gain shaping algorithm — SISO

It can directly shape the gain of closed-loop system's transfer functions (such as *S* and *T*) using the control strategy of mixed sensitivity functions in H$_\infty$ control theory, this is a problem of H$_\infty$ norm optimization, the designer has to do experiments repeatedly and performs a large number of mathematical calculations in selecting weighting functions, there is no shortcut to go, so reference [2] called the method of selection of weighting function and adjustment of its parameters an art.

The authors present a type of simplified H_∞ mixed sensitivity functions algorithm through observing the singular value curves of *S*/*T* mixed sensitivity functions (see Fig.1) and the correlativity between *S* and *T*. In Fig.1, the closed-loop frequency spectrum of a typical control system has a low pass characteristics to guarantee the robust stability, and the largest singular value equals 1 for following the reference signal *r* without steady-state error; The frequency bandwidth determines the control performance of the system, while the slope of high frequency asymptote of the frequency spectrum determines how much the system is sensitive to the frequency outside the valid frequency bandwidth, i.e. the disturbance frequency, the larger the slope is, the stronger the effect of disturbance rejection will be, this can enforce the system's robust performance, but if the slope is set too large, the designed controller may have a high order form, this is unfavorable to the implement of controller and the control effect is not improved a lot. In general, the slopes −20 dB/dec, −40 dB/dec and −60 dB/dec are suggested.

Let the bandwidth frequency of the closed-loop system be $1/T_1$ (It should be crossover frequency in the strict sense, and is approximately regarded as bandwidth frequency for the sake of easy analysis in the paper) [10], the closed-loop gain shaping controller of a standard feedback system in Fig.2 is solved below under 3 predetermined conditions (bandwidth frequency being $1/T_1$, the largest singular value being unity, and the high frequency asymptote slop being −20 dB/dec, −40 dB/dec and −60 dB/dec respectively.)

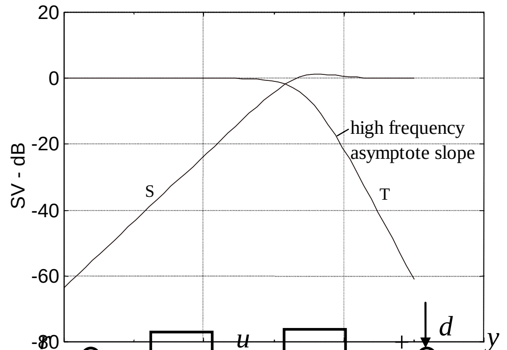


Fig.1 Typical *S*&*T* singular value curves

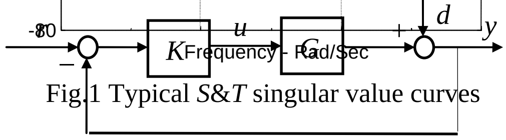


Fig.2 Configuration of a standard feedback system

(1)Controller design when the high frequency asymptote slope of the closed-loop frequency spectrum is −20 dB/dec   The frequency spectrum curve of *T* can be approximately regarded as the frequency spectrum curve of a first-order inertial system with the largest singular value of 1, then

$$T = \frac{1}{T_1 s + 1} = \frac{GK}{1 + GK}$$

$$K = \frac{1}{G T_1 s} \tag{1}$$

From the formula of *K* it follows that *K* has an integral term to guarantee no steady-state error of the system if *G* has no integral term.

(2)Controller design when the high frequency asymptote slope of the closed-loop frequency spectrum is –40 dB/dec The frequency spectrum curve of *T* can be approximately regarded as the frequency spectrum curve of a second-order inertial system with the largest singular value of 1, this is equal to the condition that the damping ratio is 1 compared to a standard oscillating second-order system, so it guarantees that there is no peak value in the frequency spectrum of *T*, then

$$\frac{1}{(T_1 s+1)^2}=\frac{GK}{1+GK}$$
$$K=\frac{1}{G2T_1 s(\frac{T_1}{2}s+1)} \tag{2}$$

(3)Controller design when the high frequency asymptote slope of the closed-loop frequency spectrum is –60 dB/dec The frequency spectrum curve of *T* can be approximately regarded as the frequency spectrum curve of a third-order inertial system with the largest singular value of 1, then

$$\frac{1}{(T_1 s+1)^3}=\frac{GK}{1+GK}$$
$$K=\frac{1}{Gs(T_1^3 s^2+3T_1^2 s+3T_1)} \tag{3}$$

Analyzing formulae (1), (2) and (3), it is clear that after the bandwidth frequency $1/T_1$ is determined according to actual performance of closed-loop system, the controller designed by using the closed-loop gain shaping algorithm depends only on the controlled plant *G*, this algorithm guarantees the shape of *S* and *T*, therefore guarantees the robust performance and robust stability of the closed-loop system.

For *K* rendered from such a procedure, e.g. as shown in (1) (2) (3), *K* appears to have higher order than that of the desired closed-loop transfer function, if the system *G* has finite zeros. Then there must exist pole-zero cancellations in computing the closed-loop system. What makes even worse is that if *G* has non-minimum phase zeros (i.e. zeros on the right-half-plane), then there will be unstable pole-zero cancellations which means the system is not internally stable. For these cases reference [11] gives a special solving method using closed-loop gain shaping algorithm.

III. Closed-loop gain shaping algorithm — MIMO

For a signal following problem, the closed-loop transfer function matrix of the system from inputs to outputs is actually the compensatory sensitive matrix

$$GK(I+GK)^{-1}=T$$
$$K=G^{-1}(I-T)^{-1}T \tag{4}$$

For a MIMO system whose *G* is a square matrix, the controller *K* can be solved from (4) for a given compensatory sensitive function *T*, the diagonal elements of *T* can be set respectively as the first-order, second-order or third-order inertial components with the largest singular values of 1, however, the situation is a little complex to design the closed-loop gain shaping controller when the controlled plant is not a square matrix, because the pseudo inverse of matrix G will be involved. The solving method is given in reference [12] when G is not a square matrix.

For a two-input-two-output system, the diagonal elements of T are set as the first-order inertial components with the largest singular values of 1 while the non-diagonal elements of T are all set as 0, then matrix T is taken as follows:

$$T = \begin{bmatrix} \dfrac{1}{T_{11}s+1} & 0 \\ 0 & \dfrac{1}{T_{22}s+1} \end{bmatrix}$$

i.e. the two inputs and two outputs are regarded as to be decoupled completely.
Example: Control of compositions of a distillation column which has two inputs and two outputs will be used as an example[13,14]

$$G(s) = \frac{1}{\tau s+1}\begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix}, \qquad \begin{bmatrix} Y \\ X \end{bmatrix} = G\begin{bmatrix} L \\ V \end{bmatrix}$$

Where the time constant $\tau = 75\,\text{min}$, Y is the overhead composition, X is the bottom composition, reflux L and boilup V are manipulated inputs for composition control.
From formula (4), we get

$$K = \frac{(75s+1)}{s}\begin{bmatrix} \dfrac{39.9417}{T_{11}} & \dfrac{-31.4869}{T_{22}} \\ \dfrac{39.4315}{T_{11}} & \dfrac{-31.9971}{T_{22}} \end{bmatrix} \qquad (5)$$

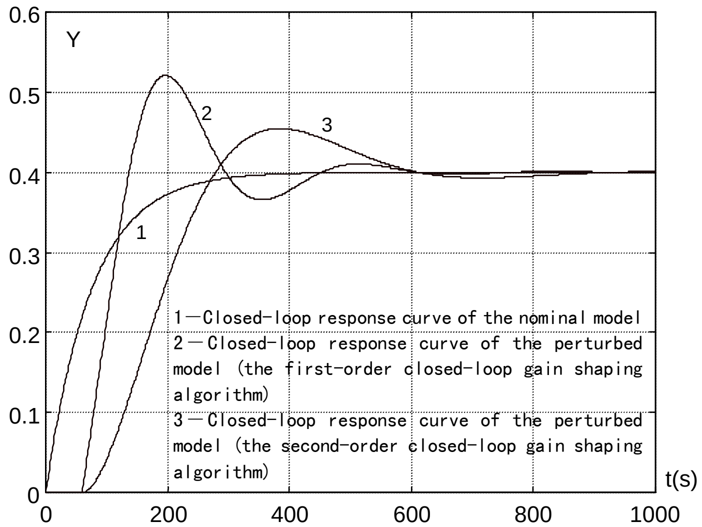
Thus K is a first-order controller taking a PI form.

Curve 1 in Fig.3 gives the simulating result of the nominal system when $T_{11} = T_{22} = 75$ and the set point values of the controlled variables X, Y are a step change vector d; curve 2 gives the closed-loop step response when the model has a pure time delay perturbation of 1 minute, from curve 2 it follows that its robustness is not satisfactory as the overshoot reaches 30% or so when the model has perturbation (1 minute). The second-order closed-loop gain shaping control algorithm is used for improving the robust performance of the system, i.e. the closed-loop transfer function matrix T is taken as:

$$T = \begin{bmatrix} \dfrac{1}{(T_{11}s+1)^2} & 0 \\ 0 & \dfrac{1}{(T_{22}s+1)^2} \end{bmatrix}$$

$$K = \frac{(75s+1)}{s}\begin{bmatrix} \dfrac{39.9417}{T_{11}} & \dfrac{-31.4869}{T_{22}} \\ \dfrac{39.4315}{T_{11}} & \dfrac{-31.9971}{T_{22}} \end{bmatrix}\begin{bmatrix} \dfrac{1}{T_{11}s+2} & 0 \\ 0 & \dfrac{1}{T_{22}s+2} \end{bmatrix} \qquad (6)$$

Curve 3 in Fig.3 gives the closed-loop step response when $T_{11} = T_{22} = 75$ and the system has a pure time delay perturbation of 1 minute, its overshoot decreases to 10% or so, the robust performance of the system is improved a lot.



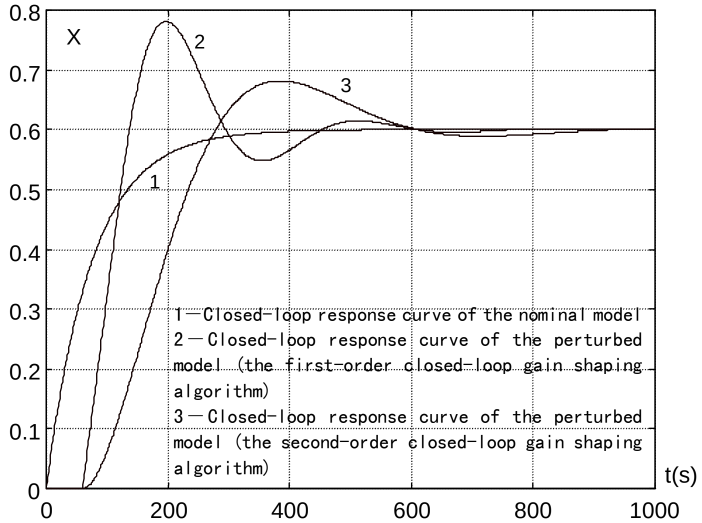


Fig.3 Step responses for closed-loop system (Input $d = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$)

The mathematical model of distillation column as an example in reference [13] is actually the same as that in reference [14], but scaling processing of variables is used in reference [13], thus the steady gain of the plant is 100 times as large as that in reference [14]. The controller designed using loop shaping algorithm of H_∞ robust control in reference [13] is

$$K = \frac{(75s+1)}{100s}\begin{bmatrix} \dfrac{39.9417}{1.42857} & \dfrac{-31.4869}{1.42857} \\ \dfrac{39.4315}{1.42857} & \dfrac{-31.9971}{1.42857} \end{bmatrix} \tag{7}$$

If the expression of right side of formula (5) is divided by 100 and takes $T_{11} = T_{22} = 1.42857$, then formula (5) is completely identical to formula (7). It follows that the closed-loop gain shaping algorithm can get the same result with loop shaping algorithm of H_∞ robust control through a simple solving process, furthermore the closed-loop gain shaping algorithm can give a better result by selecting a second-order form presented in formula (6).

IV. Conclusions

Closed-loop gain shaping algorithm is successfully applied to the control of a high purity distillation column, satisfactory control effect and robust stability are acquired. The control effect of loop shaping controller (7) given in reference [13] is the same as that of the first order closed-loop gain shaping controller in this paper, reference [13] reviews that its shortcoming is its poor robust performance of closed-loop system, this disadvantage can be overcome by using the second or the third order closed-loop gain shaping controller to increase the high frequency asymptote slope of the closed-loop frequency spectrum *T* thus cutting down the sensitivity to high frequency disturbance. The advantages of using closed-loop gain shaping algorithm to control a high purity distillation column are its simple design procedure and clear physical meaning.

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