Quadratic Stabilization of Linear Uncertain Systems with Multiple Time-Varying Delays in State and Input*

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Abstract

The problem of quadratic stabilization of linear uncertain systems with multiple time-varying delays in state and input is investigated in this paper. The parametric uncertainties are assumed to be time-varying and unknown but norm-bounded. Applying Lyapunov functional method and the notion of quadratic stabilization, sufficient conditions for designing a feedback control law to stabilize a class of uncertain linear system are presented in terms of algebraic matrix equations. The results obtained depend on the solution of a certain algebraic matrix equation respectively. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.
Keyword: quadratic stabilization; time-varying delay system; time-varying norm-bounded uncertainty; algebraic matrix equation.

I. Introduction

Quadratic stabilization of uncertain delay systems has been a classical problem for which a variety of solutions exists, see[1-25] and the references therein. These solutions differ in how uncertainty and delays are described in the model and in the tools that are used to tackle the robust analysis and synthesis problems. In [1]-[4], the uncertainties in the system is assumed to satisfy the so-called "matching conditions". These conditions are only sufficient for a given uncertain stabilizable system. In fact, there are many uncertainty systems which fail to satisfy the " matching conditions " but they are stabilizable [5]. Using the concept of quadratic stability [6], [7] and [8] have studied the quadratic stabilization of uncertain linear systems. The results of [6], [7] and [8] have been extended to time-delay [9], norm-bounded uncertainties [10], and time-varying system [11]. The stabilization problem of uncertain systems with state-delay has been studied in [12] and [13]. The work [13] has focused on the time systems with state-delay and norm-bounded time-varying uncertainties.

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In this paper, we extend the result of [13] to a class of continuous systems with norm-bounded time-varying uncertainties and multiple delays in state and input, using the concept of quadratic stabilization [6]. The system studied here fails to satisfy the matching condition. We assume parametric uncertainties to be time-varying and unknown but norm-bounded. Applying Lyapunov functional method and algebraic matrix equations, we get the conditions for designing a feedback control law to stabilize a class of uncertain linear system with multiple time-varying delays in state and input. The results depend on the solution of a certain algebraic matrix equation respectively. Finally, an example demonstrates the effectiveness of the proposed method.

Note 1. Throughout this paper, we let $\mathbf{R}=(-\infty,\infty)$, $\mathbf{R}_+=[0,\infty)$, $\mathbf{R}^n$ be any real n-dimensional linear vector space equipped with Euclidean norm $\|\cdot\|$. Given any matrix $\mathbf{W}$, let $\mathbf{W}^t$, $\mathbf{W}^{-1}$, $\lambda[\mathbf{W}]$ be, respectively, the matrix transpose, matrix inverse, and the set of eigenvalues with $\lambda_m[\mathbf{W}]$ $(\lambda_M[\mathbf{W}])$ being the minimum (maximum) eigenvalue. We use $W>o(W0)$ to denote (positive-(negative-)) definite matrix W in addition, for given $h\geq 0$, we let C denote the Branch space of continuous functions $\pi:[-h,0]\to R^n$ with

$$\|\pi\|_* := sup_{-h\leq\alpha\leq 0}\|\pi(\alpha)\| \quad (1)$$

If $X:[-h,\tau]\to R^n$ is continuous and $\tau>0$ then we introduce $X_t\in C$ and

$$x_t(\alpha)=x(t+\alpha),-h\leq\alpha\leq 0 \quad (2)$$

This means that for fixed $k\in[0,\tau]$, x_t denotes the restriction of X to the interval $[k-h,k]$ translated to $[-h,0]$. we also use the superscript "T" and "-1" represent the transpose and matric inverse respectively, $\lambda[Q]$ denotes minimum eigenvalue of matrix Q.

II. System Description

Consider a class of linear continuous time uncertain system with state-control delays

$$\dot{x}(t)=[A_0+\Delta A_0(t)]x(t)+\sum_{i=1}^{n}[A_i+\Delta A_i(t)]x(t-d_i(t))+[B_0+\Delta B_0(t)]u(t) + Du(t-e(t)) \quad (3)$$

where $x(t)\in R^n$ is the instantaneous state vector; $u(t)\in R^m$ is the control input. $A_0\in R^{n\times n}$, $A_i\in R^{n\times n}(i=1,\ldots,n), B_0, D\in R^{n\times m}$ are known constant matrices. The matrices $\Delta A_0(\cdot),\Delta A_i(\cdot)(i=1,\ldots,n)$ and $\Delta B_0(\cdot)$ are real-valued functions representing time-varying parameter uncertainties and $d_i(t)$ $(i=1,\ldots,n)$; $e(t)$ are any time-varying bounded functions satisfying

$$0\leq d_i(t)\leq d_i^*<\infty;\dot{d}_i(t)\leq\eta_i<1;0\leq e(t)\leq e^*<\infty;\dot{e}(t)\leq\eta'<1(i=1,\ldots,n) \quad (4)$$

The initial condition of system (3) is given by

$$x(t_0+\theta)=\phi(\theta),\forall\theta\in[-\max\{d_i^*,e^*\},0](i=1,\ldots,n)$$

where $\phi(\cdot)$ is a differentiable vector-value initial function defined over Banach space $C[-\max\{d_i^*,e^*\},0]$.

In this note, we assume that the uncertainties satisfy:

$$\Delta A_i(\cdot)=H_iF(t)E_i(i=0,1,\ldots,n);\Delta B_0(\cdot)=H_{n+1}F(t)E_{n+1} \quad (5)$$

Where $H_i(i=0,1,\ldots,n+1)\in R^{n\times s}, E_i(i=0,1,\ldots,n)\in R^{q\times n}, E_{n+1}\in R^{q\times m}$ are known constant matrices. $F(t)\in R^{s\times q}$ is an unknown matrix function satisfying

$$F^T(t)F(t)\leq I \quad (6)$$

with the elements of $F(\cdot)$ being Lebesgue measurable. In order to facilitate further development, we introduce

$$\begin{cases} A(t) = A_0 + H_0 F(t) E_0, B(t) = B_0 + H_{n+1} F(t) E_{n+1}, \\ C_i(t) = A_i + H_i F(t) E_i, y_i(t) = x(t - d_i(t)), \\ z(t) = x(t - e(t)), \theta_i = \left\| E_i [(1-\eta_i) R_i]^{-1} E_i^T \right\|, \\ \hat{R} = \text{diag}[(1-\eta_1) R_1 \dots (1-\eta_n) R_n \ (1-\eta^{'}) M] \end{cases} \tag{7}$$

for $i = 1, \dots, n$.

Definition 1. *The uncertain linear system (3)-(7) is said to be quadratically stabilizable if there exists a linear feedback control law* $u(t) = Kx(t)$*, positive-definite symmetric matrices* $P; R_i$ $(i = 1, \dots, n); M \in R^{m \times m}$ *and a constant* $\xi > 0$ *such that the following condition holds. Given any admissible uncertainty* $F(\cdot)$*, choosing*

$$V(t) = x^T(t) P x(t) + \sum_{i=1}^{n} \int_{t-d_i(t)}^{t} x^T(\tau) R_i x(\tau) d\tau + \int_{t-e(t)}^{t} x^T(\tau) M x(\tau) d\tau \tag{8}$$

as the Lyapunov function of closed-loop system. The Lyapunov derivative corresponding to the resulting closed-loop system satisfies the bound

$$\begin{aligned} \dot{V}(t) &= x^T(t)[PA + A^T P + 2PBK + M + \sum_{i=1}^{n} R_i] x(t) + \sum_{i=1}^{n} [2x^T(t) P C_i y_i(t) \\ &+ 2x^T(t) PDKz(t) - (1 - \dot{d}(t)) y_i^T(t) R y_i(t) - (1 - \dot{e}(t)) z^T(t) M z(t)] \\ &\le -\xi \| x(t) \|^2 \end{aligned} \tag{9}$$

for all $(x, t) \in R^{n+1}$.

Before moving on, we introduce a lemma, which is essential for the development of our result:

Lemma 1. [9] *Given any constant* $\rho > 0$ *and matrices* $\hat{D}, E, F(t)$ *with compatible dimensions such that* $F^T(t) F(t) \le I$*, then*

$$2x^T(t) P \hat{D} F(t) E x(t) \le \rho x^T(t) P \hat{D} \hat{D}^T P x(t) + \frac{1}{\rho} x^T(t) E E^T x(t) \tag{10}$$

for all $x(t) \in R^n$.

III. Main Results

In this section, we give a control law to stabilize the uncertain linear system of (3)-(7) and summarize the results as the following theorems.

Theorem 1. *Let* $S \in R^{m \times m}, Q \in R^{n \times n}, R_i \in R^{n \times n} (i = 1, \dots, n)$ *and* $M \in R^{n \times n}$ *be positive-definite symmetric matrices and suppose there exists a constant* $\rho > 0$ *such that the algebraic matrix equation*

$$\begin{aligned} &PA_0 + A_0^T P - 2PB_0 S^{-1} B_0^T P + M + \sum_{i=1}^{n} R_i + \sum_{i=1}^{n} PA_i [(1-\eta_i) R_i]^{-1} A_i^T P \\ &+ PDS^{-1} B_0^T P [(1-\eta') M]^{-1} PB_0 S^{-1} D^T P + PGP + \frac{1}{\rho} E_0^T E_0 + Q = 0 \end{aligned} \tag{11}$$

has a positive-definite symmetric solution P*, where*

$$G = \rho[\sum_{i=0}^{n+1} H_i H_i^T] + \sum_{i=1}^{n} \theta_i [H_i H_i^T] + \frac{1}{\rho}[B_0 S^{-1} E_{n+1}^T E_{n+1} S^{-1} B_0^T + \sum_{i=1}^{n} A_i [(1-\eta_i)R_i]^{-1} E_i^T E_i [(1-\eta_i)R_i]^{-1} A_i^T] \tag{12}$$

Then the uncertain delay systems (3)-(7) is quadratically stabilizable. At the same time, a suitable stabilizing control law is given by

$$u(t) = -S^{-1} B_0^T P x(t) = Kx(t). \tag{13}$$

Proof. Suppose that the algebraic matrix equation (10) has a solution $P = P^T > 0$, and let $u(t)$ be given by (12). Therefore, according to (7), the closed-loop uncertain system (3) can be expressed by

$$\begin{aligned} \dot{x}(t) &= [A_0 + H_0 F(t) E_0] x(t) + \sum_{i=1}^{n} [A_i + H_i F(t) E_i] y_i(t) \\ &\quad + [B_0 + H_{n+1} F(t) E_{n+1}] Kx(t) + DKz(t) \\ &= [A(t) + B(t)K] x(t) + \sum_{i=1}^{n} C_i(t) y_i(t) + DKz(t) \end{aligned} \tag{14}$$

For the Lyapunov function

$$V(t) = x^T(t) P x(t) + \sum_{i=1}^{n} \int_{t-d_i(t)}^{t} x^T(\tau) R_i x(\tau) d\tau + \int_{t-e(t)}^{t} x^T(\tau) M x(\tau) d\tau.$$

which is positive-definite for all $x(t) \neq 0$, the Lyapunov derivative corresponding to the closed- loop system (14) is given by

$$\begin{aligned} \dot{V}(t) &= x^T(t)[PA + A^T P + 2PBK + M + \sum_{i=1}^{n} R_i] x(t) + \sum_{i=1}^{n} [2x^T(t) P C_i(t) y_i(t) \\ &\quad + 2x^T(t) PDKz(t) - (1-\dot{d}_i(t)) y_i^T(t) R_i y_i(t) - (1-\dot{e}(t)) z^T(t) M z(t)] \end{aligned} \tag{15}$$

We can rewrite (13) into the following form

$$\dot{V}(t) = [x^T(t) \quad y_1^T(t) \quad \ldots \quad y_n^T(t) \quad z^T(t)] \begin{bmatrix} \Xi & PC_1 & \cdots & PC_n & PDK \\ C_1^T P & -(1-\dot{d}_1(t))R_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_n^T P & 0 & \cdots & -(1-\dot{d}_n(t))R_n & 0 \\ K^T D^T P & 0 & \cdots & 0 & -(1-\dot{e}(t))M \end{bmatrix} \begin{bmatrix} x(t) \\ y_1(t) \\ \vdots \\ y_n(t) \\ z(t) \end{bmatrix} \tag{16}$$

where

$$\Xi = PA + A^T P + 2PBK + M + \sum_{i=1}^{n} R_i$$

By using Schur complement argument, it following from (16) that

$$\begin{aligned} W &= PA + A^T P + PBK + M + \sum_{i=1}^{n} R_i + K^T B^T P + \begin{bmatrix} PC_1 & \ldots & PC_n & PDK \end{bmatrix} \hat{R}^{-1} \\ &\quad \begin{bmatrix} PC_1 & \ldots & PC_n & PDK \end{bmatrix}^T = PA + A^T P + M + \sum_{i=1}^{n} R_i + K^T B^T P + PBK \\ &\quad + \sum_{i=1}^{n} PC_i [(1-\eta_i)R_i]^{-1} C_i^T P + PDK[(1-\eta')M]^{-1} K^T D^T P < 0 \end{aligned} \tag{17}$$

In order to verifying (16), we should construct an upper bound for the Lyapunov derivative in (17). So, we rewrite W into the following way

$$\begin{aligned} x^T(t)Wx(t) = x^T(t)\sum_{i=1}^{n}\{[PA_0 + A_0{}^T P - 2PB_0S^{-1}B_0{}^T P + M + R_i \\ + PA_i[(1-\eta_i)R_i]^{-1}A_i{}^T P + PDK[(1-\eta')M]^{-1}K^T D^T P] \\ + [PH_0FE_0 + E_0{}^T F^T H_0{}^T P] \\ + [PH_{n+1}FE_{n+1}K + K^T E_{n+1}{}^T F^T H_{n+1}{}^T P] \\ + [PA_i[(1-\eta_i)R_i]^{-1}E_i{}^T F^T H_i{}^T P + PH_iFE_i[(1-\eta_i)R_i]^{-1}A_i{}^T P] \\ + [PH_iFE_i[(1-\eta_i)R_i]^{-1}E_i{}^T F^T H_i{}^T P]\}x(t) \end{aligned} \tag{18}$$

Applying (13) to the second, third, forth and last terms in (18) we can get

$$\begin{aligned} x^T(t)[PH_0FE_0 + E_0{}^T F^T H_0{}^T P]x(t) = 2x^T(t)PH_0FE_0x(t) \\ \le \rho x^T(t)PH_0H_0{}^T Px(t) + \frac{1}{\rho}x^T(t)E_0{}^T E_0x(t) \end{aligned} \tag{19}$$

$$\begin{aligned} x^T(t)[PH_{n+1}FE_{n+1}K + K^T E_{n+1}{}^T F^T H_{n+1}{}^T P]x(t) = 2x^T(t)PH_{n+1}FE_{n+1}Kx(t) \\ \le \rho x^T(t)PH_{n+1}H_{n+1}{}^T Px(t) + \frac{1}{\rho}x^T(t)K^T E_{n+1}{}^T E_{n+1}Kx(t) \end{aligned} \tag{20}$$

$$\begin{aligned} & x^T(t)[PA_i[(1-\eta_i)R_i]^{-1}E_i{}^T F^T H_i{}^T P + PH_iFE_i[(1-\eta_i)R_i]^{-1}A_i{}^T P]x(t) \\ & = 2x^T(t)PA_i[(1-\eta_i)R_i]^{-1}E_i{}^T F^T H_i{}^T Px(t) \\ & \le \rho x^T(t)PH_iH_i{}^T Px(t) \\ & + \frac{1}{\rho}x^T(t)[PA_i[(1-\eta_i)R_i]^{-1}E_i{}^T E_i[(1-\eta_i)R_i]^{-1}A_i{}^T P]x(t) \end{aligned} \tag{21}$$

$$x^T(t)PH_iFE_i[(1-\eta_i)R_i]^{-1}E_i{}^T F^T H_i{}^T Px(t) \le \theta_i x^T(t)PH_iH_i{}^T Px(t) \tag{22}$$

So, from (19), (20), (21) and (22) we can obtain

$$\dot{V}(t) \le -x^T(t)Qx^T(t) \le -\lambda_m[Q]\|x(t)\|^2 \tag{23}$$

For all $(x,t) \in R^n \times R$. Therefore the inequality (6) is satisfied with $\xi = \lambda_m[Q] > 0$. □

Remark 1. In system (3), if $D = 0$, the system becomes the system discussed in [15]. In nominal system (without uncertainties), we can see that $H_0 = 0, E_0 = 0, H_i = 0, E_i = 0, H_{n+1} = 0, E_3 = 0,$ and hence $G \equiv 0$. It turns out that the quadratic stability requires

$$\begin{aligned} PA_0 + A_0{}^T P - 2PB_0S^{-1}B_0{}^T P + \sum_{i=1}^{n} PA_i[(1-\eta_i)R_i]^{-1}A_i{}^T P \\ + PDS^{-1}B_0{}^T P[(1-\eta')M]^{-1}PB_0S^{-1}D^T P + M + \sum_{i=1}^{n} R_i < 0 \end{aligned} \tag{24}$$

Remark 2. It should be emphasized that the upper bound is not conservative. A larger upper bound can be obtained by the following theorem.

Theorem 2. *Let* $S \in R^{m\times m}, Q \in R^{n\times n}, R_i \in R^{n\times n} (i = 1,\dots,n), M \in R^{m\times m}$ *be given positive-definite symmetric matrices and suppose that there exists a constant* $\rho > 0$ *such that the algebraic matrix equation (8) has a definite solution.*

Then given any positive-definite symmetric matrices $S^* \in R^{m\times m}, Q^* \in R^{n\times n}$, *and* $R_i^* \in R^{n\times n}$, $M^* \in R^{m\times m}$, *there exists a constant* $\rho^* > 0$. *Such that given any* $\tilde{\rho} \in [0, \rho^*]$, *the algebraic matrix equation*

$$PA_0 + A_0^T P - 2PB_0(S^*)^{-1}B_0^T P + M^* + \sum_{i=1}^{n} R_i^* + \sum_{i=1}^{n} PA_i[(1-\eta_i)R_i^*]^{-1}A_i^T P + PD(S^*)^{-1}$$

$$\times B_0^T P[(1-\eta')M^*]^{-1} PB_0(S^*)^{-1}D^T P + PG^* P + \frac{1}{\tilde{\rho}} E_0^T E_0 + Q^* = 0 \qquad (25)$$

where

$$G^* = \tilde{\rho}[\sum_{i=0}^{n+1} H_i H_i^T] + \sum_{i=1}^{n} \theta_i [H_i H_i^T] + \frac{1}{\tilde{\rho}}[B_0(S^*)^{-1}E_{n+1}^T E_{n+1}(S^*)^{-1}B_0^T$$

$$+ \sum_{i=1}^{n} A_i[(1-\eta_i)R_i^*]^{-1} E_i^T E_i [(1-\eta_i)R_i^*]^{-1} A_i^T]$$

has a positive-definite symmetric solution P.

Proof. Suppose that P is a positive-definite symmetric solution to (10). Therefore $P^* = \rho P$ satisfies the algebraic matrix equation

$$P^* A_0 + A_0^T P^* - \frac{2}{\rho} P^* B_0 (S^*)^{-1} B_0^T P^* + \rho M^* + \sum_{i=1}^{n} \rho R_i^* + \sum_{i=1}^{n} \frac{1}{\rho} P^* A_i [(1-\eta_i)R_i^*]^{-1} A_i^T P^*$$

$$+ \frac{1}{\rho^3} P^* D(S^*)^{-1} B_0^T P^* [(1-\eta')M^*]^{-1} P^* B_0 (S^*)^{-1} D^T P^* + \frac{1}{\rho} P^* G^* P^* + E_0^T E_0 + \rho Q^* = 0$$

Using the results of [7] and [14], it directly follows that there exists a constant $\rho^* > 0$ such that the algebraic matrix equation

$$\tilde{P} A_0 + A_0^T \tilde{P} - \frac{2}{\hat{\rho}} \tilde{P} B_0 (S^*)^{-1} B_0^T \tilde{P} + \hat{\rho} M^* + \sum_{i=1}^{n} \hat{\rho} R_i^* + \sum_{i=1}^{n} \frac{1}{\hat{\rho}} \tilde{P} A_i [(1-\eta_i)R_i^*]^{-1} A_i^T \tilde{P}$$

$$+ \frac{1}{\hat{\rho}^3} \tilde{P} D(S^*)^{-1} B_0^T \tilde{P}[(1-\eta')M^*]^{-1} \tilde{P} B_0 (S^*)^{-1} D^T \tilde{P} + \frac{1}{\hat{\rho}} \tilde{P} G^* \tilde{P} + E_0^T E_0 + \hat{\rho} Q^* = 0$$

has a positive-definite symmetric solution $\tilde{P}$ for all $\hat{\rho} \in (0, \rho^*]$. □

Remark 3. We can see from Theorem 2 that the failure or success of the algorithm is independent of the selection of Q, S, and $R_i (n = 1,\dots,n)$. If the algorithm succeeds, we can conclude that the uncertain system (1)-(5) is quadratically stabilization by control law (10).

IV. Simulation Example

Now, we consider a dynamic system with the following data

$$A_0 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0.1 \\ -0.2 & -0.3 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \Delta A_0(t) = \begin{bmatrix} 0 & 0.2\sin t \\ 0 & 0.1\sin t \end{bmatrix},$$

$$\Delta A_1(t) = \begin{bmatrix} 0.2\sin t & 0 \\ 0.1\sin t & 0 \end{bmatrix}, \quad \Delta B_0(t) = \begin{bmatrix} 0.2\sin t \\ 0.1\sin t \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \phi = \begin{bmatrix} 2 \\ -1 \end{bmatrix},$$

$$d_1(t) = \mu|\cos(\omega t)|, \quad e(t) = \nu|\cos \varpi t|, \quad F(t) = \sin t, \quad E_1 = [0 \quad 1], \quad E_2 = [1 \quad 0],$$

$$E_3 = 1, \quad M = R_1 = Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad H_0^{\ T} = H_1^{\ T} = H_2^{\ T} = [0.2 \quad 0.1], \quad \rho = 0.01,$$

$$\mu = \nu = 9, \eta = \eta' = 0.9.$$

We can get from Matlab

$$P = \begin{bmatrix} 2.3325 & -0.0239 \\ -0.0239 & 0.4473 \end{bmatrix}, \quad \|P\| = 2.3328, \quad \theta = 10.$$

So the feedback control law can be chosen as

$$u(t) = [0.0239 \quad -0.4473]x(t).$$

The resulting closed-loop state trajectory is presented in figure 1.

V. Conclusion

This paper has established a linear feedback control law that can stabilize a uncertain system with state-control delay and time-varying unknown-but-bounded parameters. The controller in this paper is obtained from solving an algebraic matrix equation. An simulation example illustrates the effectiveness of the method that given in this paper.

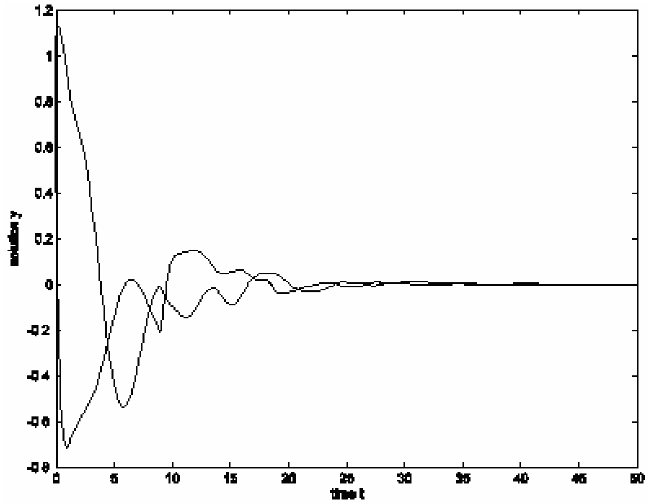

Fig. 1. Quadratic stabilization of uncertain delay system

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