Estimate of parameter-deviation degree for delayed neural networks

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Abstract

The stability of a neural network model may often be destroyed by the parameter deviations during the implementation. However, few results (if any) for the asymptotical stability of such system with a certain degree of parameter deviations have been reported in the literature. In this paper, we present a simple delayed neural network model, in which each parameter deviates the reference point with the degree no more than a certain value, and further investigate the robust asymptotical stability of this model and estimate the maximum permissible deviation degree.

Keyword: Neural networks, time delays,

I. Introduction

During recent decades, several neural network models with or without delays have been extensively studied, particularly, regarding their stability analysis [1-8]. However, It is well known that the stability of a given system may often be destroyed by its unavoidable uncertainty due to the existence of modeling error, external disturbance and parameter fluctuation during the implementation. So it is essential to introduce the robust technique to design a system with such uncertainty. If the uncertainty of a system is only due to the deviations and perturbations of its parameters, and if these deviations and perturbations are all bounded, then the system is called an interval system. Recently, several global and robust stability criteria for interval neural networks with constant or time-varying delays have been proposed [9, 10-13]) since the pioneering work of Liao and Yu [10]. In most existing literature about the uncertain neural network model, the maximum permissible deviation degree of the system parameters has not been investigated and estimated. However, this is an unavoidable and important factor to be considered during the implementation. In this paper, we use functional differential equation to describe a neural network model with parameter deviation, and further study its robust stability with respect to the parameter deviation degree.

For this purpose, consider a neural network with parameter deviations described by

$$\dot{u}(t) = -Au(t) + W\,g(u(t-\tau)) + I \tag{1}$$

where $u(t) = [u_1(t), \ \cdots \ ,u_n(t)]^T$ is the neuron state vector. $A = diag(a_1, a_2, \cdots, a_n)$ is a positive diagonal matrix, $g(u) = [g_1(u_1), \ \cdots \ ,g_n(u_n)]^T$ denotes the neuron activation functions with $g(0) = 0$, $I = [I_1, \ \cdots \ ,I_n]^T$ is a constant vector, $W = (w_{ij})_{n\times n}$ is the connect-weighting matrix, $\tau > 0$ is transmission delay. Throughout this paper, we assume that the permissible deviation degree of the system parameters is denoted by $\alpha \ge 0$. Here, if we select the matrices $A_0 = diag(a_i^{(0)})_{n\times n} > 0$ (throughout this paper, we use $\Omega > 0$ to denote the symmetrical positive matrix Ω) and $W_0 = (w_{ij}^{(0)})_{n\times n}$ as the reference matrices of *A* and *W*, respectively, then $A \in N[A_0, \alpha]$ and $W \in N[W_0, \alpha]$, where

$$N[A_0,\alpha] = \left\{ diag(a_i)_{n\times n} \,\middle|\, (1-\alpha)a_i^{(0)} \le a_i \le (1+\alpha)a_i^{(0)} \right\},$$

and

$$N[W_0,\alpha] = \left\{ (w_{ij})_{n\times n} \,\middle|\, (1-\alpha)w_{ij}^{(0)} \le w_{ij} \le (1+\alpha)w_{ij}^{(0)} \text{ or } (1+\alpha)w_{ij}^{(0)} \le w_{ij} \le (1-\alpha)w_{ij}^{(0)} \right\}$$

Furthermore, we assume that each activation function in (1) is bounded and satisfies the following sector condition: There exists a positive constant, $k > 0$, such that

$$0 \le \frac{g_j(x) - g_j(y)}{x - y} \le k \text{ , for any } x, y \in R, j = 1, 2, \cdots, n. \tag{2}$$

Due to the boundedness of the activation functions, there exists at least an equilibrium point u^* for system (1). In the following, we always shift this equilibrium point into the origin. By making the transformation $x(t) = u(t) - u^*$, we convert model (1) to the following:

$$\dot{x}(t) = -A\,x(t) + W\, f(x(t-\tau)) \tag{3}$$

where $f_j(x_j(t)) = g_j(x_j(t) + u_j^*) - g_j(u_j^*)$, $j = 1, 2, \cdots, n$. Note that f_j also satisfies a sector condition in the form of

$$f(x_j(t))[f(x_j(t)) - k\,x_j] \le 0 \tag{4}$$

II. Stability Analysis

In this section, model (3) is first transformed into another equivalent form, which allows us to analyze expediently. And then a simple robust stability condition for model (3) will be derived.

For this purpose, let

$$E_W = \left[\sqrt{|w_{11}^{(0)}|}e_1 \quad \cdots \quad \sqrt{|w_{1n}^{(0)}|}e_1 \quad \cdots \quad \sqrt{|w_{n1}^{(0)}|}e_n \quad \cdots \quad \sqrt{|w_{nn}^{(0)}|}e_n \right]_{n\times n^2},$$

and

$$F_W^T = \left[\sqrt{\left|w_{11}^{(0)}\right|} e_1 \quad \cdots \quad \sqrt{\left|w_{1n}^{(0)}\right|} e_n \quad \cdots \quad \sqrt{\left|w_{n1}^{(0)}\right|} e_1 \quad \cdots \quad \sqrt{\left|w_{nn}^{(0)}\right|} e_n \right]_{n \times n^2},$$

where e_i denotes the *i*-th column-vector of the $n \times n$ identity matrix. Obviously, we have

$$E_W E_W^T = diag\left(\sum_{j=1}^{n} \left|w_{1j}^{(0)}\right| \quad \cdots \quad \sum_{j=1}^{n} \left|w_{nj}^{(0)}\right| \right),$$
$$F_W^T F_W = diag\left(\sum_{j=1}^{n} \left|w_{j1}^{(0)}\right| \quad \cdots \quad \sum_{j=1}^{n} \left|w_{jn}^{(0)}\right| \right). \tag{5}$$

Furthermore, let

$$\Sigma^* = \left\{ \Sigma \in R^{n^2 \times n^2} \middle| \Sigma = diag(\varepsilon_{11}, \cdots, \varepsilon_{1n}, \cdots, \varepsilon_{n1}, \cdots \varepsilon_{nn}), \left|\varepsilon_{ij}\right| \le 1 \right\} \tag{6}$$

Then, we have the following result, which can be proved directly via simple matrix operation.

Lemma 1. Let $M[W_0, \alpha] = \left\{ W = W_0 + \alpha^2 E_W \Sigma_W F_W \middle| \Sigma_W \in \Sigma^* \right\}$. Then, $M[W_0, \alpha] = N[W_0, \alpha]$.

From Lemma 1, the model (3) can be rewritten as

$$\dot{x}(t) = -A x(t) + \left(W_0 + \alpha^2 E_W \Sigma_W F_W\right) F(x(t - \tau)) \tag{7}$$

where $A \in N[A_0, \alpha]$ and $\Sigma_W \in \Sigma^*$.

Obviously, system (1) or (3) is of the same stability property as system (7). It is also worth noting that the uncertain matrices A and Σ_W are both diagonal, and, in particular, each diagonal element of Σ_W is not larger than one in absolute value. Thus, the transfer above may lead to a new and better approach to analyze the robust stability issues of model (1). To state our theoretical result, the following lemma is useful.

Lemma 2. [14] Given any real matrices $\Sigma_1, \Sigma_2, \Sigma_3$ of appropriate dimensions and a scalar $\varepsilon > 0$ such that $0 < \Sigma_3 = \Sigma_3^T$. Then, the following inequality holds:

$$\Sigma_1^T \Sigma_2 + \Sigma_2^T \Sigma_1 \le \varepsilon \Sigma_1^T \Sigma_3 \Sigma_1 + \varepsilon^{-1} \Sigma_2^T \Sigma_3^{-1} \Sigma_2.$$

Theorem 1. Suppose that there exist positive constants $\beta > 0$, $\gamma > 0$ and $0 \le \overline{\alpha} < 1$ such that

$$D - \frac{1}{\beta} W_0 W_0^T > 0 \tag{8}$$

where $D = diag(d_1, d_2, \cdots, d_n)$ with

$$d_i = \frac{(1 - \overline{\alpha}) a_i^{(0)}}{k} - \beta - \overline{\alpha}^2 \left(\frac{1}{\gamma} \sum_{j=1}^{n} \left|w_{ij}^{(0)}\right| + \gamma \sum_{j=1}^{n} \left|w_{ji}^{(0)}\right| \right), \; i = 1, 2, \cdots, n.$$

Then model (7) is robust asymptotically stable at the origin for any $0 \le \alpha \le \overline{\alpha}$.

Proof. Select a Lyapunov function as

$$V(x(t)) = \sum_{j=1}^{n} \int_0^{x_j} f_j(s) ds + \int_{t-\tau}^{t} f^T x(\xi) Q f(x(\xi)) d\xi \tag{9}$$

where $Q = \beta E + \alpha^2 \gamma F_W^T F_W > 0$ $(0 \le \alpha \le \overline{\alpha})$ and E denotes an identity matrix.

Using the method presented in [2] and its references, it is easy to verify that (9) is a Lyapunov function. The time derivative of $V(x(t))$ along the trajectories of (7) is

$$\dot{V}(x(t)) = f^T(x(t))\dot{x}(t) + f^T(x(t))Q\,f(x(t)) - \mu\, f^T(x(t-\tau))Q\,f(x(t-\tau))$$
$$= -f^T(x(t))Ax(t) + f^T(x(t))W_0\,f(x(t-\tau)) + f^T(x(t))\alpha^2 E_W \Sigma_W F_W\, f(x(t-\tau))$$
$$+ f^T(x(t))Q\,f(x(t)) - f^T(x(t-\tau))Q\,f(x(t-\tau)).$$

By Eq. (4) and Lemma 2, we have

$$\dot{V}(x(t)) \le -f^T(x(t))\left[\frac{A}{k} - \frac{1}{\gamma}\alpha^2 E_W E_W^T - \beta E - \alpha^2\gamma\, F_W^T F_W - \frac{1}{\beta}W_0 W_0^T\right] f(x(t))$$

$$\le -f^T(x(t))\left[diag\left(\frac{(1-\alpha)a_i^{(0)}}{k} - \beta - \alpha^2\left(\frac{1}{\gamma}\sum_{j=1}^{n}\left|w_{ij}^{(0)}\right| + \gamma\sum_{j=1}^{n}\left|w_{ji}^{(0)}\right|\right)\right) - \frac{1}{\beta}W_0 W_0^T\right] f(x(t)).$$

Obviously, $D - \frac{1}{\beta}W_0 W_0^T > 0$ implies

$$diag\left(\frac{(1-\alpha)a_i^{(0)}}{k} - \beta - \alpha^2\left(\frac{1}{\gamma}\sum_{j=1}^{n}\left|w_{ij}^{(0)}\right| + \gamma\sum_{j=1}^{n}\left|w_{ji}^{(0)}\right|\right)\right) - \frac{1}{\beta}W_0 W_0^T > 0,$$

which follows $\dot{V}(x(t)) < 0$. The proof thus completed.

Because D is a positive diagonal matrix and $W_0 W_0^T$ is a symmetrical matrix, the theorem above can be easily reduced to the following corollary.

Corollary 1. Let $\lambda_{max}(M)$ denote the largest eigenvalue of matrix M. Suppose that there exist positive constants $\beta > 0$, $\gamma > 0$ and $0 \le \overline{\alpha} < 1$ such that

$$\lambda_{max}\left(W_0 W_0^T\right) < \beta \min_{1 \le i \le n}\{d_i\} \qquad \textbf{(10)}$$

where d_i is defined in Theorem 1. Then, model (7) is robust asymptotically stable at the origin for any $0 \le \alpha \le \overline{\alpha}$.

In many neural networks, particularly in biological models, the connection-weight matrix is symmetrical, i.e., $W_0^T = W_0$ in (7), which implies $\sum_{j=1}^{n}\left|w_{ij}^{(0)}\right| = \sum_{j=1}^{n}\left|w_{ji}^{(0)}\right|$, for any $i = 1, 2, \cdots, n$. Therefore, it is easy to see that $\frac{1}{\gamma}\sum_{j=1}^{n}\left|w_{ij}^{(0)}\right| + \gamma\sum_{j=1}^{n}\left|w_{ji}^{(0)}\right| = \left(\frac{1}{\gamma} + \gamma\right)\sum_{j=1}^{n}\left|w_{ij}^{(0)}\right| \ge 2\sum_{j=1}^{n}\left|w_{ij}^{(0)}\right|$. Selecting $\gamma = 1$ in Corollary 1 and furthermore $\beta = \sqrt{\lambda_{max}\left(W_0 W_0^T\right)}$, we have the following result.

Corollary 2. Suppose that $W_0^T = W_0$ and there exists $0 \le \overline{\alpha} < 1$ such that

$$\sqrt{\lambda_{max}\left(W_0 W_0^T\right)} < \min_{1 \le i \le n}\left\{\frac{(1-\overline{\alpha})a_i^{(0)}}{2k} - \overline{\alpha}^2\sum_{j=1}^{n}\left|w_{ij}^{(0)}\right|\right\} \qquad \textbf{(11)}$$

Then model (7) is robust asymptotically stable at the origin for any $0 \le \alpha \le \overline{\alpha}$.

From the theoretical analysis above, we can see that the network (3) is always stable only if the deviation degree of each parameter is not larger than $\overline{\alpha}$ and that the same degree for all the parameters is not required. Although we can derive some less conservative stability conditions for model (7), we use only the above results in this paper, because they are very simple and easy to verify. Furthermore, from these conditions, we can estimate easily the maximum permissible deviation degree $\overline{\alpha}$ by solving Eq. (11).

III. Example

To illustrate our results we consider model (1) with sigmoid function:

$$f_i(x) = 0.5(\,|\,x+1\,|-|\,x-1\,|\,),\ i = 1,2,\cdots,n.$$

Furthermore, for notational and computational convenience, we consider only a neural network with four neurons. The reference matrices are taken as

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } W_0 = \begin{bmatrix} 0 & 0.05 & 0.03 & -0.06 \\ 0.05 & 0 & 0.1 & 0.02 \\ 0.03 & 0.1 & 0 & -0.04 \\ -0.06 & 0.02 & -0.04 & 0 \end{bmatrix}.$$

From Corollary 3, we calculate that the maximum permissible deviation degree is $\overline{\alpha} = 0.76394$. That is to say, the model (1) is always stable at an equilibrium as long as the deviation degree of the parameters of model (1) is not larger than 0.76394. Figs. 1 and 2 show the convergence behaviors for this system with different parameter matrices, respectively.

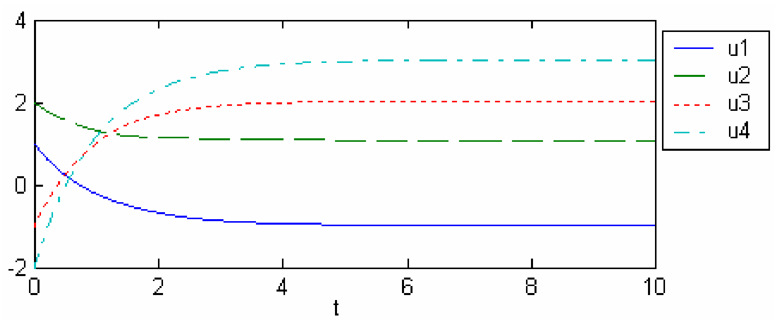

Fig. 1. The time response curves for the reference system with $\tau = 1$, the initial conditions $u(\theta) = [1 \;\; 2 \;\; -1 \;\; -2]^T$, for any $\theta \in [-\tau, 0]$, the external input $I = [-1 \;\; 1 \;\; 2 \;\; 3]^T$.

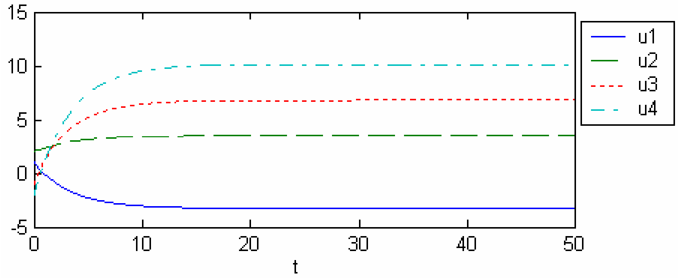

Fig. 2. The time response curves for the neural network (1) with the parameter matrices: $A=(1-0.7)A_0$, W is constructed as follows: the 1st and 4th rows are same as those of W_0, the 2nd and 3rd rows are (1-0.3) and (1+0.5) times of those of W_0, respectively, $\tau=1$, the initial conditions $u(\theta)=[1 \quad 2 \quad -1 \quad -2]^T$, for any $\theta\in[-\tau,0]$, the external input $I=[-1 \quad 1 \quad 2 \quad 3]^T$.

IV. Conclusions

We have presented a simple delayed neural network with parameter deviation and analyzed its robust stability, particularly, estimated the maximum permissible deviation degree using the proposed stability conditions. Because the issue investigated in this paper may often arise during the implementation of neural networks, the results here are practical and should be useful for further studies of this kind of neural network model.

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