An Annealing Recurrent Neural Network for Extremum Seeking Control

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Abstract

The application of sinusoidal periodic search signals results in the “chatter” problem of the output and the switching of the control law of the general extremum seeking control (ESC). An annealing recurrent neural network (RNN) is proposed for ESC to solve those problems in the general ESC. The paper converts ESC into seeking the extreme point where the slope of Cost Function is zero, and applies an annealing recurrent neural network to finding the point and stabilizing the plant at the point. ESC combined with the annealing RNN doesn’t make use of search signals such as sinusoidal periodic signals, which solves those problems in previous ESC and improves the dynamic performance of the ESC system greatly. At the same time, it can be simplified by the proposed method to analyze the stability of ESC. The simulation results of a simplified UAV tight formation flight model proved the advantages mentioned above.
Keyword: Recurrent Neural Network, Extremum Seeking Control, UAV, Tight Formation Flight.

I. Introduction

A. *Background*

Early work on performance improvement by extremum seeking can be found in Tsien. In the 1950s and 1960s, ESC was considered as an adaptive control method[1]. Until 1990s sliding mode control for extremum seeking has not been utilized successfully[2]. Subsequently, a method of adding compensator dynamics in ESC was proposed by Krstic, which improved the stability of the system[3]. Although those methods improved tremendously the performance of ESC, the “chatter” problem of the output and the switching of the control law limit the application of ESC.

B. *Main Contribution*

The method of combining an annealing recurrent neural network with ESC is proposed in the paper. First, this paper converts ESC into seeking the extreme point where the slope of cost function is zero; second, constructs an annealing RNN; then, applies the annealing RNN to finding the extreme point and stabilizing the plant at the point. The annealing RNN proposed in the paper doesn’t make use of search signals such as sinusoidal periodic signals, so the method can solve the “chatter” problem of the output and the switching of the control law in the general ESC and improve the dynamic performance of the ESC system, which are validated by simulating a simplified tight formation flight model consisting of two Unmanned Aerial

Vehicles. At the same time, it can be simplified by the proposed method to analyse the stability of ESC.

II. Problem Formulation

Consider a single-input single-output nonlinear system:

$$\begin{aligned} \dot{x} &= f\left(x(t), u(t)\right) \\ y &= F\left(x(t)\right) \end{aligned} \quad \textbf{(1)}$$

Where $x \in R, u \in R$ and $y \in R$ are the state, the control and the output variables, respectively. $F(x)$ is also defined as the cost function of the system. $f(x,u)$ and $F(x)$ are smooth functions[4].

Assumption 1: There is a smooth control law:

$$u(t) = \alpha\left(x(t), \theta\right) \quad \textbf{(2)}$$

to stabilize the nonlinear system(1), where θ is a parameter which determines a unique equilibrium point.

With the control law (2), the closed-loop system of the nonlinear system (1) can be written as:

$$\dot{x} = f\left(x, \alpha(x, \theta)\right)$$

Assumption 2: There is a smooth function $x_e : R \to R^n$ such that:

$$f\left(x, \alpha(x, \theta)\right) = 0 \leftrightarrow x = x_e(\theta)$$

Assumption 3: The static performance map at the equilibrium point $x_e(\theta)$ from θ to y represented by:

$$y = F\left(x_e(\theta)\right) = F(\theta) \quad \textbf{(3)}$$

is smooth and has a unique maximum or minimum point $\theta^* \in R$ such that:

$$F'\left(\theta^*\right) = 0$$

$$\text{and } F''\left(\theta^*\right) > 0 \text{ or } F''\left(\theta^*\right) < 0$$

Differentiating (3) with respect to time yields the relation between $\dot{\theta}$ and $\dot{y}(t)$.

$$J\left(\theta(t)\right)\dot{\theta}(t) = \dot{y}(t) \quad \textbf{(4)}$$

where $J\left(\theta(t)\right) = \dfrac{\partial F(\theta)}{\partial \theta}$.

By Assumption 3, if the system converges to an extreme point θ^*, at the same time $\left|J(\theta)\right|$ will also converge to zero. An annealing recurrent neural network is applied to ESC to minimize $\left|J(\theta)\right|$ in finite time. Certainly the system is subjected to (4).

Then, the optimization problem can be written as follows.

Minimize: $\left|J(\theta)\right|$

Subject to: $J\left(\theta(t)\right)\dot{\theta}(t) = \dot{y}(t)$ **(5)**

The optimization (5) is then equivalent to

Minimize: $c^T \xi$

Subject to: $A\xi = b$ **(6)**

where, $\xi = \begin{bmatrix} J(\theta) \\ |J(\theta)| \\ \dot{\theta}(t) \end{bmatrix}$, $c = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$, $A = \begin{bmatrix} 1 & -sign(J(\theta)) & 0 \\ \dot{\theta}(t) & 0 & 0 \\ 0 & 0 & J(\theta) \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ \dot{y}(t) \\ \dot{y}(t) \end{bmatrix}$, and

$$\mathrm{sign}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} .$$

By the dual theory, the dual program corresponding to the program (6) is

Maximize: $b^T z$

Subject to: $A^T z = c$ **(7)**

where, $z = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix}^T$.

Therefore, an extremum seeking problem is converted into the programs defined in (6) and (7).

III. Problem Formulation

C. Energy Function

In view of the primal and dual programs (6) and (7), define the following energy function:

$$E(\xi,z) = \frac{1}{2}\left(c^T\xi - b^Tz\right)^2 + \frac{1}{2}\left\|T(t)\left(A\xi - b\right)\right\|^2 + \frac{1}{2}\left\|T(t)\left(A^Tz - c\right)\right\|^2 \qquad \mathbf{(8)}$$

Clearly, the energy function (8) is convex and continuously differentiable. The first term in (8) is the squared difference between the objective functions of the programs (6) and (7), respectively. The second and the third terms are for the equality constraints of (6) and (7). $T(t)$ is a time-varying annealing parameter matrix, $T = diag\left(\eta_1 e^{-\beta_1 t}, \eta_2 e^{-\beta_2 t}, \eta_3 e^{-\beta_3 t}\right)_{3\times 3}$, $\eta_i, \beta_i \ (i = 1,2,3)$ are positive scalar constants, which are used to scale the annealing rate.

D. Dynamic Equations

With the energy function defined in (8), the dynamics for the neural network solving (6) and (7) can be defined by the negative gradient of the energy function as follows:

$$\frac{dv}{dt} = -\mu \nabla E(v) \qquad \mathbf{(9)}$$

where, $v = (\xi, z)^T$, $\nabla E(v)$ is the gradient of the energy function $E(v)$ defined in (8), and μ is a positive scalar constant, which is used to scale the convergence rate of the recurrent neural network.

The dynamical equation (9) can be expressed as:

$$\frac{d\xi}{dt} = -\mu\left[c\left(c^T\xi - b^Tz\right) + A^T T^T(t) T(t)\left(A\xi - b\right)\right] \qquad \mathbf{(10)}$$

$$\frac{dz}{dt} = -\mu\left[-b\left(c^T\xi - b^Tz\right) + A T^T(t) T(t)\left(A^Tz - c\right)\right] \qquad \mathbf{(11)}$$

E. Neural Network Architecture

The architecture of the recurrent neural network is shown as follows:

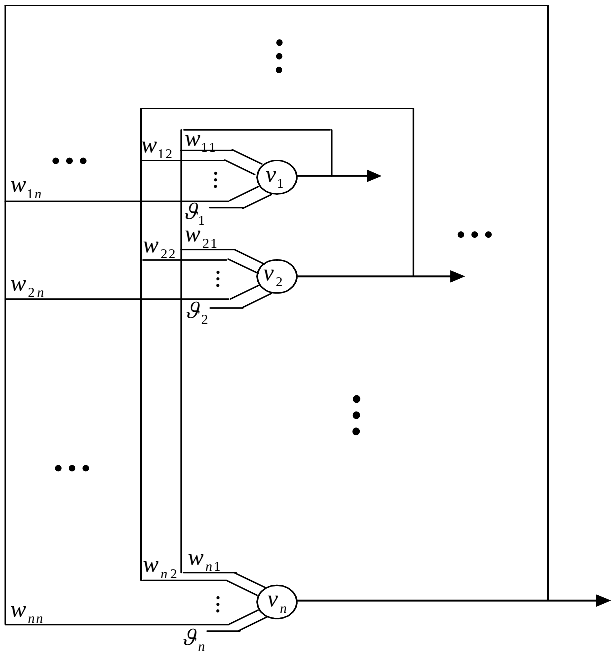

Fig. 1. the architecture of the recurrent neural network

Equations (10) and (11) can be converted into the equations as follows:

$$\begin{aligned}
&\frac{du_1}{dt} = -\mu\left(cc^T + A^T T^T(t) T(t) A\right)v_1 + \mu b^T v_2 + \mu A^T T^T(t) T(t) b \\
&\frac{du_2}{dt} = \mu b c^T v_1 - \mu\left(bb^T + AT^T(t) T(t) A^T\right)v_2 + \mu A T^T(t) T(t) c \\
&v_1 = u_1 \\
&v_2 = u_2
\end{aligned} \tag{12}$$

The annealing neural network consists of 6 lateral connected neurons, as shown in figure 1, which is determined by the number of decision variables such as (ξ, z), (u_1, u_2) is a column vector of instantaneous net inputs to neurons, (v_1, v_2) is a column output vector, and equals to (ξ, z). The lateral connection weight matrix is defined as $\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} = \begin{bmatrix} -\mu\left(cc^T + T(t)A^T A\right) & \mu b^T \\ \mu b c^T & -\mu\left(bb^T + T(t) A A^T\right) \end{bmatrix}$, the biasing threshold vector of the neurons is defined as $\begin{bmatrix} \vartheta_1 \\ \vartheta_2 \end{bmatrix} = \begin{bmatrix} \mu A^T T^T(t) T(t) b \\ \mu A T^T(t) T(t) c \end{bmatrix}$. By adjusting μ, η and β, the weight matrix and the biasing threshold vector can be adjusted. Therefore, the annealing RNN control process is shown as figure 2.

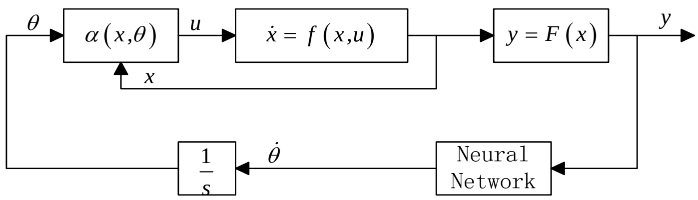

Fig.2. Block diagram of the annealing RNN control process

IV. Convergence Analysis

We analyze the stability of the proposed annealing RNN controller in the section.

Lemma 1[5]: Suppose that $f: D \subset R^n \to R$ is differentiable on a convex set $D_0 \subset D$. Then f is convex on D_0 if and only if

$$(z-y)^T \nabla f(y) \le f(z) - f(y), \ \forall y, z \in D_0 \qquad \textbf{(13)}$$

where $\nabla f(y)$ is the gradient of $f(y)$.

Lemma 1 has been proved at length in [5].

Lemma 2: The optimal solution to the programs (6) and (7) are ξ^* and z^*, respectively, if and only if $E(v^*) = 0$ and

$$\left(v^* - v\right)^T \nabla E(v,t) \le -E(v,t) \qquad \textbf{(14)}$$

where $v^* = \left(\xi^{*T}, z^{*T}\right)^T$ and $v = \left(\xi^T, z^T\right)^T$.

Proof: Form the definition of the energy function (8), it can easily find that $E(v^*) = 0$ if and only if v^* is the optimal solution of (6) and (7). Since for all v, the energy function $E(u,t) \ge 0$ is continuously differentiable and convex. Therefore, we have the conclusion of the Lemma 2 from Lemma 1.

Theorem: The annealing recurrent neural network defined in (12) is globally stable and converges to the optimal solutions of the program (6) and (7).

Proof: Without loss of generality, let $\mu = 1$. Consider the following Lyapunov function:

$$V(v) = \frac{1}{2}\left(v^* - v\right)^T \left(v^* - v\right) \qquad \textbf{(15)}$$

Where $v^* = \left(\xi^{*T}, z^{*T}\right)^T$, and ξ^*, z^* are the optimal solutions to the programs (6) and (7), respectively. From Lemma 2 and the equation (9), we have

$$\frac{dV}{dt} = \frac{dV}{dv}\left(\frac{dv}{dt}\right) = \left(v^* - v\right)^T \frac{dv}{dt} = \left(v^* - v\right)^T \nabla E(v) \le -E(v) \le 0 \qquad \textbf{(16)}$$

Since $E(v) \ge 0$, according to the Lyapunov's theorem theory, the neural network defined in (12) is Lyapunov stable. From Lemma 2, $E(v^*) = 0$ if and only if $\nabla E(v^*) = 0$. Hence v^* makes $\dot{v} = 0$ and $\dot{V} = 0$, and therefore the neural network defined in (12) converges to its equilibrium points, and then $|J(\theta)|$ converges to its minimum point. So $\theta = \theta^*$, the output y of the system (1) equals to the optimal solution $y^* = F(\theta^*)$.

Since $E(v,t)$ is continuously differentiable and convex for all v, the local minimum is equivalent to the global minimum. The annealing RNN defined in (12) is thus globally stable and converges to the optimal solutions of the programs (6) and (7). The proof is completed.

V. Simulation Results

Consider a simplified tight formation flight model consisting of two Unmanned Aerial Vehicles[6].

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -20 & -9 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -35 & -15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \textbf{(17)}$$

with a cost function given by

$$y(t) = -10\left(x_1(t)+0\right)^2 - 5\left(x_3(t)+9\right)^2 + 590 \quad \textbf{(18)}$$

where x_1 is the vertical separation of two Unmanned Aerial Vehicles, x_2 is the differential of x_1, x_3 is the lateral separation of two Unmanned Aerial Vehicles, x_4 is the differential of x_3 and y is the upwash force acting on the wingman. It is clear the maximum point is $x_1^* = 0$ and $x_3^* = -9$ where the cost function $y(t)$ reaches its maximum $y^* = 590$.

A control law based on sliding mode theory is given by:

$$\begin{cases} \sigma_1 = s_1 x_1 + x_2 \\ u_1 = 20x_1 + (9 - s_1)x_2 - k_1 sign(\sigma_1 - s_1\theta_1) \\ \sigma_2 = s_2 x_3 + x_4 \\ u_2 = 35x_3 + (15 - s_2)x_4 - k_2 sign(\sigma_2 - s_2\theta_2) \end{cases} \quad \textbf{(19)}$$

where σ_1, σ_2 are two sliding mode surfaces, s_1, k_1, s_2, k_2 are positive scalar constants, θ_1, θ_2 are two extremum seeking parameters, which an annealing recurrent neural network is used to seek at the same time.

Remark: The control law is given in (19), which is based on sliding mode theory. We choose $sign(\sigma_i - s_i\theta_i)$ $(i = 1,2)$ so that x_1 and x_3 entirely traces θ_1 and θ_2 in the sliding mode surfaces respectively, and the system will be stable at θ_1^* and θ_2^* finally.

Applying the annealing recurrent neural network to the system (17), the initial conditions are given as $x_1(0) = -2$, $x_2(0) = 0$, $x_3(0) = -4$, $x_4(0) = 0$, $\theta_1(0) = -2$, $\theta_2(0) = -4$ and the parameters: $\eta_{11} = 1.0$, $\eta_{12} = 1.0$, $\eta_{13} = 0.03$, $\beta_{11} = 0$, $\beta_{12} = 0$, $\beta_{13} = 0.01$, $\eta_{21} = 1.0$, $\eta_{22} = 1.0$, $\eta_{23} = 0.09$, $\beta_{21} = 0$, $\beta_{22} = 0$, $\beta_{23} = 0.01$, $s_1 = 2.15$, $s_2 = 3.35$, $k_1 = k_2 = 1$, $\mu_1 = 0.235$, $\mu_2 = 0.019$, respectively. The simulation results are shown from figure 3 to figure 9.

Where, $\mu_i\,(i = 1,2)$ are the main factors of scaling the convergence rate of the recurrent neural network, if they are too big, the error of the output will be introduced; on the contrary, if they are too small, the convergence rate of the system will be slow. The range of $\eta_{ij}\,(i = 1,2;\, j = 1,2,3)$ is $[0,1]$, the corresponding η_{ij} of the seeking object $J(\theta)$ ought to approach 1. $\beta_{ij}\,(i = 1,2;\, j = 1,2,3)$ are the main factors of scaling the annealing rate of $T(t)$, the values of β_{ij} should not be too big, otherwise the system will be unstable. Certainly, the values of those parameters should be verified by the system simulation.

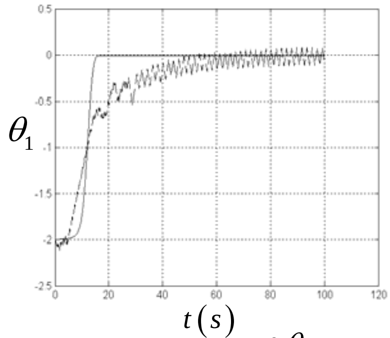

Fig. 3. the result of θ_1

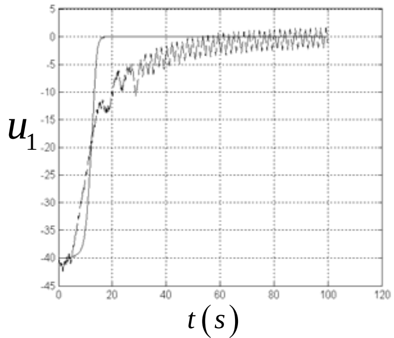

Fig. 4. the control law u_1

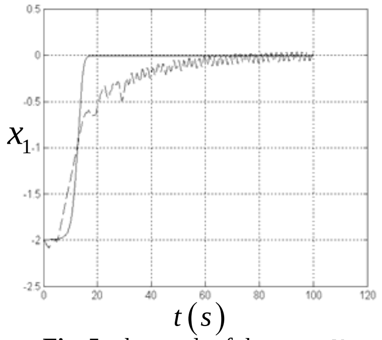

Fig. 5. the result of the state x_1

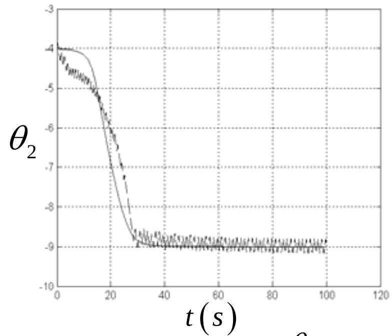

Fig. 6. the result of θ_2

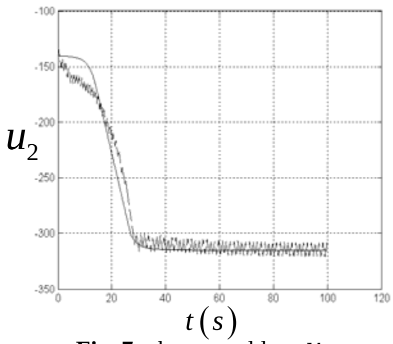

Fig. 7. the control law u_2

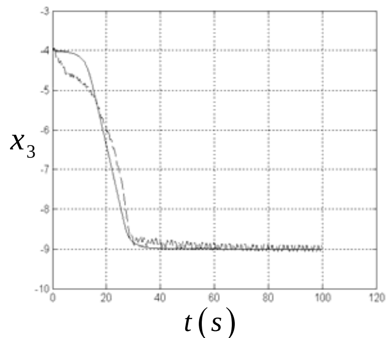

Fig. 8. the result of the state x_3

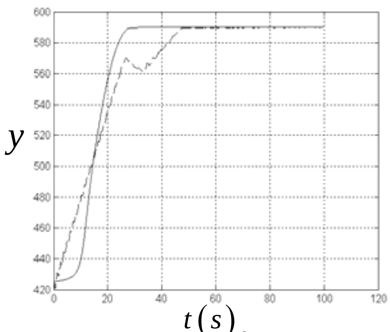

Fig. 9. the result of the output y

Where, solid lines are the results applying ESC with the annealing RNN; dash lines are the results applying ESC with sliding mode[7]. Comparing those simulation results, we know the dynamic performance of the method proposed in the paper is better than that of ESC with sliding mode. The "chatter" of the output doesn't exist in figure 4 and 7, which is very harmful in practice. Moreover the convergence rate of ESC with the annealing RNN can be scaled by adjusting the weight matrix.

VI. Conclusion

The method of combining an annealing recurrent neural network with ESC improves the dynamic performance of the system greatly. At the same time, the disappearance of the "chatter" of the system output and the switching of the control law are beneficial to engineering applications.

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