Fuzzy Sliding-Mode Controller Design Based on W-stability Theorem

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Abstract

In this paper, base on a new conclusion W-stability theorem, a novel adaptive fuzzy neural networks(FNN) controller is designed for some BTT missile integrated with sliding-mode control(SMC). Like second order sliding mode control, this control scheme can not only guarantee the convergence of the sliding surface but also the derivation of it. Besides, the W-stability theorem is used to prove the stability of the system and to deduce the adaptive laws of the FNN. When the adaptive FNN is used to approximate the uncertainty part of the perturbed nonlinear terms, the gain of the SMC part will be more proper, as a result, the chattering effect can be alleviated. Finally, the correctness and effectiveness of the method have been verified by simulation using nonlinear overload model of pitch channel of some BTT missile.
Keyword: W-stability theorem SMC FNN BTT missile.

I. Introduction

FNNs combines the low-level learning and computational power of neural networks with high-level reasoning and decision making of fuzzy systems[1].The FNN constructs can be classified into two categories: 1)fuzzy neural structures based on Takagi-Sugeno inference method with crisp consequent functions[2] and 2) fuzzy neural structures of fuzzy rules and fuzzy consequents. A familiar member of the first category have the architecture which employs a fixed fuzzy rule base and performing only parameter learning [3]. The FNN used in this paper are proposed by M. Onder. Efe[4] before, and this time we will consult to W-stability theorem to improve the stability of the total system and deduce the adaptive laws of the FNN.
It is known to all that to analysis the internal stability, the basic tool is Lyapunov functions. For the input-output stability , a new type of input-output stability (W-stability) is defined in [5,6], a new result will be got, when the stability theorem is used for the FNN. The uncertainty of model is a problem that we can hardly avoid in nonlinear control, performance of many control systems is limited by variations of system parameters and external disturbances. Sliding-mode control (SMC)[7-10] is often a favored control approach, because the insensitivity property toward the parametric uncertainties and the external disturbances. But the discontinuous control often causes chattering effect, which is usually caused by the use of sign function of sliding surface. To solve this problem, many scholars combine fuzzy control with sliding-mode control, it is what we call fuzzy sliding-mode control (FSMC)[11,12]. In order to alleviate the chattering effect to some extent, the

FNN is used to approximate equivalent control part of the nonlinear system and SMC is used to deal with the approximating error part [13].

The paper is organized as the follows, in section 2, the FNN to be used is introduced, in section 3, the W-stability theorem is introduced, in section 4, the mathematical model and designing process of FNN and SMC is discussed, and in section 5, an example is given and some simulation results are presented, finally a conclusions is drawn.

II. Description of FNN

The FNN adopted in this paper is described as follows, which uses bell member functions as (1)

$$\mu_{ij}(u_j) = 1/(1+\left|\frac{u_j - c_{ij}}{a_{ij}}\right|^{2b_{ij}}) \tag{1}$$

Where c_{ij} presents the center of the member function, a_{ij} determine the width of the bell function and b_{ij} characterize the slope. The following rule sets are adopted,

$$\begin{aligned}&IF\ \ u_1\ \ is\ \ U_1\ \ AND \cdots\ u_m\ \ is\ \ Um\\ &THEN\ F = y_i \qquad\qquad i = 1,2,..R\end{aligned} \tag{2}$$

where u and F are input and output of the fuzzy logic , U_i and ξ_i are input and output linguistic variables. We chose product-operation rule of fuzzy implication and center of average deffuzifier as shown in (3)

$$F_{\mathrm{TOTAL}} = \sum_{i=1}^{R}\xi_i\prod_{j=1}^{m}\mu_{ij}(u_j)/\sum_{i=1}^{R}\prod_{j=1}^{m}\mu_{ij}(u_j) = \sum_{i=1}^{R}\xi_i\omega_{ni} = \xi^{\mathrm{T}}\omega_n \tag{3}$$

Where

$$\omega_{ni} = \prod_{j=1}^{m}\mu_{ij}(u_j)/\sum_{i=1}^{R}\prod_{j=1}^{m}\mu_{ij}(u_j) \tag{4}$$

IV. Introduction of W-Stability Theorem

If the norm of any vector x in R^n is denoted as $\|x\|$, $W_{1,2}^n$ denotes the Sobolev space of functions $x: R^+ \to R^+$, then x and its distributional-derivative $\dot{x}$ belong to L_2^n. If the inner product of two functions x and y in $W_{1,2}^n$ is defined as(5), Lemma 1 to be introduced will become understandable.

$$\langle x, y\rangle_W = \langle x, y\rangle + \langle \dot{x}, \dot{y}\rangle \tag{5}$$

Lemma 1[14]: Consider the systems(6)

$$\begin{aligned}&x^{(n)} = f(x,\dot{x},...,x^{(n-1)}) + g(x,\dot{x},...,x^{(n-1)})u + d\\ &y = h(x,\dot{x},...,x^{(n-1)})\end{aligned} \tag{6}$$

where $x \in R$, $y \in R$, $u \in R$, and suppose that f and g are uniformly Lipschitz continuous with respect to its second and third arguments in its domain and $g(x) \le m_1 \|x\|$. Suppose that

1) There is a Lyapunov function V that satisfies

$$c_1 \|x\|^2 \le V(x,t) \le c_2 \|x\|^2 \quad (7)$$

$$\partial V / \partial t + (\partial V / \partial x) f \le c_3 \|x\|^2 \quad (8)$$

$$\left\| \frac{\partial V}{\partial x} \right\| \le c_4 \|x\|^2 \quad (9)$$

for all $x \in R$, and for positive constants c_1, c_2, c_3, c_4, h is continuous differential to all its arguments, and the norm of $\frac{\partial h}{\partial x}$ is bounded by a constant c ,then the system is $g - W - s$ when $m_1 < c_1 c_3 /(c_2 c_4)$.

IV. Problem Formulation and Design of FNN Controller

The design process of the controller is demonstrated for a typical SISO nonlinear system, the form of which the pitch overload model of some BTT missile can be changed into.Consider the nonlinear SISO system as follows

$$\begin{aligned} x^{(n)} &= f(x, \dot{x}, \ldots, x^{(n-1)}) + g(x, \dot{x}, \ldots, x^{(n-1)})u + d \\ y &= x \end{aligned} \quad (10)$$

First, the error variable is defined,

$$e = y - y_d \quad (11)$$

where, y_d is the desired output, the sliding manifold is chosen as (12)

$$S = e^{(n-1)} + k_1 e^{(n-2)} + \ldots + k_{n-1} e \quad (12)$$

according to W-stability theorem the candidate Lyapunov function is chosen as (13)

$$V_W = \frac{1}{2} < S, S >= \frac{1}{2}(S^2 + \dot{S}^2) \quad (13)$$

According to Lyapunov stability theory, we seek a control to enable $\dot{V}_W < 0$, so that the asymptotical stability of the system is guaranteed. Because the resulting control u is a sign function of S, the chattering of the system is unavoidable.
Let the nominal model of (10) be

$$\begin{aligned} x^{(n)} &= \bar{f}(x, \dot{x}, \ldots, x^{(n-1)}) + \bar{g}(x, \dot{x}, \ldots, x^{(n-1)})u \\ y &= x \end{aligned} \quad (14)$$

We choose the sliding manifold as (12), let $\dot{S} = 0$, and then we can get the equivalent control

$$u_{eq} = \frac{1}{\bar{g}(.)}[-\bar{f}(.) + y_d^{(n)} - K^{\mathrm{T}}E] \tag{15}$$

where, $K = (k_{n-1},...,k_1)^{\mathrm{T}}$, $E = (\dot{e},...,e^{(n-1)})^{\mathrm{T}}$ .For $f(.) = \bar{f}(.) + \Delta f(.)$, $g(.) = \bar{g}(.) + \Delta g(.)$ and the existence of the disturbance d , the equivalent control u_{eq} cannot guarantee the stability of the system, so that an additional control u_f is needed to offset the influence of the disturbance and the parameter variations. Before designing the controller, the following assumptions are made.

Assumption 1: Equivalent control u_{eq}, FNN control u_f and fuzzy member function are bounded and Lipschitz continuous with respect to time, and S_0, the initial value of S is bounded, namely

$$\|S_0\| < \theta_S,\ \|u_{eq}\| < \theta_{u_{eq}},\ \|\dot{u}_{eq}\| < \theta_{\dot{u}_{eq}},\ \|\xi\| < \theta_\xi,\ \|\dot{\omega}_n\| < \theta_{\dot{\omega}_n},\ \|\omega_n\| < \theta_{\omega_n} \tag{16}$$

Assumption 2: There exist upper bound and lower bound of $g(.)$,besides, Δf , Δg and d are Lipschitz continuous with respect to time, namely

$$\gamma_{g(.)} < \|g(.)\| < \theta_{g(.)},\ \|\Delta\dot{g}(.)\| < \theta_{\Delta\dot{g}(.)},\ \|\Delta\dot{f}(.)\| < \theta_{\Delta\dot{f}(.)},\ \|\dot{d}\| < \theta_{\dot{d}} \tag{17}$$

Let the total control be $u = u_{eq} + u_f$, according to (10), the following equation can be obtained

$$\begin{aligned} x^{(n)} &= \bar{f}(.) + \Delta f(.) + [\bar{g}(.) + \Delta g(.)]u + d \\ &= \bar{f}(.) + \bar{g}(.)u_{eq} + \Delta f(.) + \Delta g(.)u_{eq} + [\bar{g}(.) + \Delta g(.)]u_f + d \end{aligned} \tag{18}$$

then it can obtain that

$$\dot{S} = \Delta f(.) + \Delta g(.)u_{eq} + [\bar{g}(.) + \Delta g(.)]u_f + d \tag{19}$$

Let the FNN control be defined as $u_f = \xi^T\omega_n$, then (19) can be changed into (20)

$$\begin{aligned} \dot{S} &= \Delta f(.) + \Delta g(.)u_{eq} + (\bar{g}(.) + \Delta g(.))\xi^{\mathrm{T}}\omega_n + d \\ &= \Delta f(.) + \Delta g(.)u_{eq} + g(.)\xi^T\omega_n + d \end{aligned} \tag{20}$$

According to W-stability theorem, choosing the candidate Lyapunov function as (13), taking its derivative, it can obtain that

$$\begin{aligned} \dot{V}_W &= S\dot{S} + \dot{S}\ddot{S} = \dot{S}[S + \Delta\dot{f}(.) + \Delta\dot{g}(.)u_{eq} + \Delta g(.)\dot{u}_{eq} \\ &\quad + \dot{g}(.)\xi^{\mathrm{T}}\omega_n + g(.)\dot{\xi}^{\mathrm{T}}\omega_n + g(.)\xi^{\mathrm{T}}\dot{\omega}_n + \dot{d}] \end{aligned} \tag{21}$$

let

$$\dot{\xi} = -k\omega_n / \omega_n{}^T\omega_n sign(\dot{S}) \tag{22}$$

then we have

$$\begin{aligned}
\dot{V} &= \dot{S}[S + \Delta\dot{f}(.) + \Delta\dot{g}(.)u_{eq} + \Delta g(.)\dot{u}_{eq} + \dot{g}(.)\xi^{\mathrm{T}}\omega_n - kg(.)sign(\dot{S}) + g(.)\xi^{\mathrm{T}}\dot{\omega}_n + \dot{d}] \\
&= -kg(.)\left|\dot{S}\right| + \dot{S}[S + \Delta\dot{f}(.) + \Delta\dot{g}(.)u_{eq} + \Delta g(.)\dot{u}_{eq} + \dot{g}(.)\xi^{\mathrm{T}}\omega_n + g(.)\xi^{\mathrm{T}}\dot{\omega}_n + \dot{d}] \\
&< -k\gamma_{g(.)}\left|\dot{S}\right| + \left|\dot{S}\right|[\theta_S + \theta_{\Delta\dot{f}(.)} + \theta_{\Delta\dot{g}(.)}\theta_{u_{eq}} + \theta_{\Delta g(.)}\theta_{\dot{u}_{eq}} + \theta_{\dot{g}(.)}\theta_\xi\theta_{\omega_n} + \theta_{g(.)}\theta_\xi\theta_{\dot{\omega}_n} + \theta_{\dot{d}}] \\
&= -(k\gamma_{g(.)} - \theta_S - \theta_{\Delta\dot{f}(.)} - \theta_{\Delta\dot{g}(.)}\theta_{u_{eq}} - \theta_{\Delta g(.)}\theta_{\dot{u}_{eq}} - \theta_{\dot{g}(.)}\theta_\xi\theta_{\omega_n} - \theta_{g(.)}\theta_\xi\theta_{\dot{\omega}_n} - \theta_{\dot{d}})\left|\dot{S}\right|
\end{aligned} \tag{23}$$

if the condition of (24) can be satisfied

$$k > (\theta_S + \theta_{\Delta\dot{f}(.)} + \theta_{\Delta\dot{g}(.)}\theta_{u_{eq}} + \theta_{\Delta g(.)}\theta_{\dot{u}_{eq}} + \theta_{\dot{g}(.)}\theta_\xi\theta_{\omega_n} + \theta_{g(.)}\theta_\xi\theta_{\dot{\omega}_n} + \theta_{\dot{d}})/\gamma_{g(.)} \tag{24}$$

then (25) can be guaranteed

$$\dot{V}_W < 0 \tag{25}$$

According to the Lemma 1, W-stability of the system is guaranteed. The structure of the control system is shown in Figure 1.

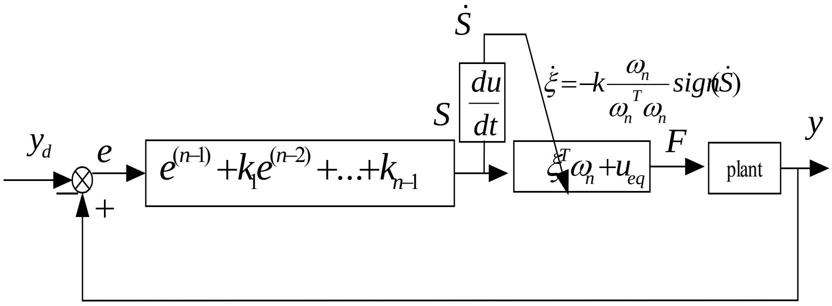

Figure 1: The block diagram of the control system

V. Simulation

To verify the correctness and the effectiveness of the approach, simulations have been made based on the pitch overload model of some BTT missile, the mathematical model of which is as follows

$$\begin{aligned}
\ddot{n}_y &= f(.) + g(.)\delta_z + d \\
y &= n_y
\end{aligned} \tag{26}$$

where, $f(.)$, $g(.)$, d, f_1, f_2 can be found in[15]. Let $e = n_y - n_{yd}$ and $S = \dot{e} + k_1 e$,choose the nominal model $\ddot{n}_y = -\dot{n}_y - 40n_y + 35\delta_z + d$ (Height=2000m, Velocity=1200 m/s), the fuzzy rules and member function are chosen as [15], Let $k = 20$ and equivalent control $u_{eq} = \frac{1}{\bar{g}(.)}[-\bar{f}(.) + y_d^{(2)} - 2\dot{e} - 10e]$. When the input is 5, the simulation result of the nominal system and the system with $\pm 30\%$ parameter disturbance are shown in figure 1. For the system with -30% parameter disturbance, the curves of the δ_z , S , $\dot{S}$ are shown in figure2-4. From the simulation results, we can see that good performance can achieved by adopting the control scheme, and the chattering effect is alleviated.

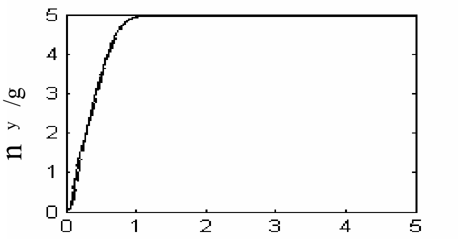

Figure 1 Output responses

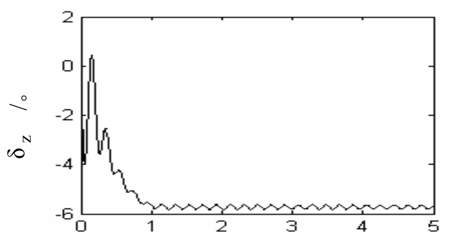

Figure 2 Control efforts

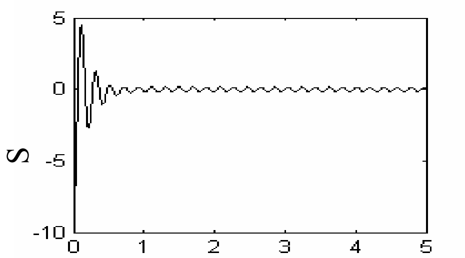

Figure 3 Trajectory of S

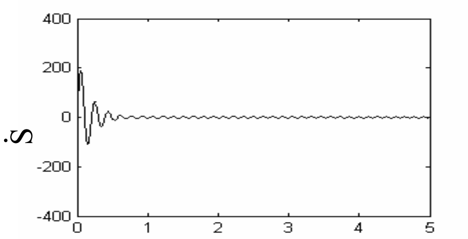

Figure 4 Trajectory of $\dot{S}$

VI. Conclusion

In this paper, a novel adaptive fuzzy neural networks(FNN) controller is designed for some BTT missile integrated with sliding-mode control(SMC). The adaptive FNN is used to approximate the uncertainty part of the perturbed nonlinear terms, such that the gain of the SMC part will be proper, and the chattering effect can be alleviated. Besides, the W-stability theorem is used to prove the stability of the system and the adaptive laws are deduced. Finally, the correctness and effectiveness of the method have been verified by simulation using nonlinear overload model of pitch channel of some BTT missile. However, seen from the simulation results there is still a little chattering when the system output reaches stable state, how to alleviate the chattering more effectively is our work in the future.

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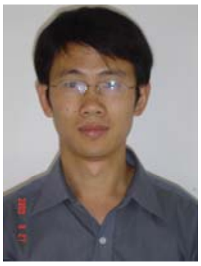

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