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Design of Reconfiguring Control Systems via State Feedback Eigenstructure Assignment

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Abstract

In this paper the design of reconfiguring a class of linear control systems via state feedback eigenstructure assignment is investigated. The design aim is to resynthesize a state feedback control law such that the eigenvalues of the reconfigured closed-loop control system can completely recover those of the original close-loop system, and make the corresponding eigenvectors of the former as close to those of the latter as possible. General complete parametric expressions for the state feedback gains are established in term of a set of parametric vectors and the closed-loop poles. The set of parametric vectors and the set of closed-loop poles represent the degrees of freedom existing in the reconfiguring design, and can be further properly chosen to meet some desired specification requirement, such as robustness. An illustrative example and the simulation results show that the proposed parametric method is effective and simple.

Keyword: Linear control systems, eigenstructure assignment, state feedback, reconfiguration.

1 Introduction

Reconfigured Control Systems (RCS) posses the ability of accommodating system failures automatically with some prior assumptions. In recent years, RCS has drawn much attention of many researchers, and many new methods and schemes have been proposed (see, e. g. [1]-[7] and their references). In addition to linear quadratic regulator method [1], pseudo inverse method [2], inverse component-mode synthesis method [3], Lyapunov method [4] and LMI method [5], eigenstructure assignment method ([6] and [7]) becomes more and more attractive. Based on the fact that the performances of a control system are mainly determined by their eigenvalues and the corresponding eigenvectors, thus eigenstructure assignment method is convenient to redesign a new gain matrix in order to recover the eigenvalues of the normal control system and make their corresponding eigenvectors of the reconfigured closed-loop systems as close to those of the normal closed-loop system as possible. Parametric methods for eigenstructure assignment have been intensively studied in [8]-[19] for conventional linear systems, descriptor linear systems and second-order dynamic systems. The parametric methods give the parametric expressions of all the control laws and all the closed-loop eigenvector matrices. These free parametric vectors included in these expressions and the closed-loop eigenvalues, offer all the degrees of design freedom and can be further utilized to satisfy certain specifications in some control system designs.

In this paper, we will consider the design of reconfiguring linear systems via state feedback eigenstructure assignment. Based on the result for state feedback eigenstructure assignment proposed by Duan in [8], a parametric form of all the resynthesized gain matrices is derived and a corresponding algorithm for this reconfiguration is proposed. This parametric method offers all the degrees of design freedom, which can be utilized to satisfying additional performances in control system designs.

2 Problem Formulation

Consider a linear control system in the form of

$$\dot{x} = Ax + Bu , \tag{1}$$

where $x \in \mathbf{R}^n$ and $u \in \mathbf{R}^r$ are the state and input vectors, respectively; A and B are known matrices with appropriate dimensions and $\mathbf{rank}B = r$; the matrix pair (A, B) is controllable, that is,

$$\mathbf{rank}[A - sI_n \quad B] = n , \ \forall s \in \mathbf{C} . \tag{2}$$

Because of the outstanding variations, the system (1) often becomes into the following form

$$\dot{x}_f = A_f x_f + B_f u_f , \tag{3}$$

where $x_f \in \mathbf{R}^n$ and $u_f \in \mathbf{R}^m$ are the state and input vectors, respectively; A_f and B_f are known matrices with appropriate dimensions and $\mathbf{rank}B_f = m$; the matrix pair (A_f, B_f) is controllable, that is,

$$\mathbf{rank}[A_f - sI_n \quad B_f] = n , \ \forall s \in \mathbf{C} . \tag{4}$$

For convenience, we call system (1) the normal linear system and system (3) the fault linear system. Applying the following state feedback control law

$$u = Kx , \ K \in R^{r \times n} , \tag{5}$$

to the system (1), yields its closed-loop system as

$$\dot{x} = A_c x , A_c = A + BK . \tag{6}$$

Recall the fact that non-defective matrices possess eigenvalues which are less insensitive with respect to parameter perturbations, in this paper, we only consider the eigenvalues of the closed-loop system (6) are distinct and self-conjugate. Denoting the eigenvalues of system (6) by $\sigma(A_c) = \{s_i \in C, i = 1, 2, \cdots, n\}$, where s_i,

$i=1, 2, \cdots, n$, are distinct and self-conjugate and their corresponding eigenvectors by $v_i \in C^n$, $i=1, 2, \cdots, n$, produces

$$A_c v_i = s_i v_i, \ i=1, 2, \cdots, n. \tag{7}$$

Applying the following state feedback controller

$$u_f = K_f x_f, \ K_f \in R^{m\times n}, \tag{8}$$

to the fault system (3), obtains

$$\dot{x}_f = A_{fc} x_f, A_{fc} = A_f + B_f K_f. \tag{9}$$

Due to the fact that the internal behaviors of a control system are determined by its eigenvalues together with their corresponding eigenvectors, and the performances of its closed-loop system can be improved by modifying the eigenvalues and the corresponding eigenvectors with some feedback control laws, then the problem of reconfiguring the linear system (1) via state feedback to be solved in this paper can be stated as follows.

Problem RESA: Given the controllable normal system (1), its controllable fault system (3), and a set of self-conjugate distinct complex numbers s_i, $i=1, 2, \cdots, n$, then redesign a new state feedback controller (8) such that

$$\sigma(A_c)=\sigma(A_{fc})=\{s_i \in C, i=1, 2, \cdots, n\}, \tag{10}$$

and

$$J_i = \left\| v_{fi} - v_i \right\|^2, \ i=1, 2, \cdots, n, \tag{11}$$

are minimized, where v_{fi}, $v_i \in C^n$, $i=1, 2, \cdots, n$, are the eigenvectors of the closed-loop matrices A_{fc} and A_c associated with s_i, $i=1, 2, \cdots, n$.

Remark 1. From the description of Problem RESA, it is clear to see that when the relation (10) is satisfied, there hold

$$A_{fc} v_{fi} = s_i v_{fi}, \ i=1, 2, \cdots, n. \tag{12}$$

3 Closed-Loop Eigenstructure Assignment

Set

$$\Lambda = \mathrm{diag}(s_1, s_2, \cdots, s_n), \ V=[v_1 \ v_2 \cdots v_n],$$

Equation (7) is clearly reduced into the following form:

$$AV + BKV = V\Lambda. \tag{13}$$

Further, denote

$$W = KV. \tag{14}$$

then equation (13) is changed into its equivalent form:

$$AV + BW = V\Lambda. \tag{15}$$

Because the matrix pair (A, B) is controllable, applying a series of element matrix transformations to matrix $[A - sI \ \ B]$, we can obtain a pair of unimodular matrices $P(s) \in R^{n\times n}[s]$ and $Q(s) \in R^{(n+r)\times(n+r)}[s]$ satisfying the following equation:

$$P(s)\,[A - sI \quad B]\,Q(s) = [0 \quad I], \ \forall s \in \mathbf{C}. \tag{16}$$

Partition $Q(s)$ into the following form

$$Q(s) = \begin{bmatrix} Q_{11}(s) & Q_{12}(s) \\ Q_{21}(s) & Q_{22}(s) \end{bmatrix}, \ Q_{11}(s) \in R^{n\times r}[s], \tag{17}$$

then we can obtain the following lemma which gives the parametric expressions of eigenstructure assignment via state feedback in time-invariant linear systems.

Lemma 1[8] Given matrices *A* and *B* with $\mathrm{rank}(B) = r$, and a group of self-conjugate distinct complex numbers s_i, $i = 1, 2, \cdots, n$, if the matrix pair $(A,\ B)$ is controllable, then the parametric expressions of all the state feedback gain matrix K in (13) can be given by

$$K = WV^{-1}, \tag{18}$$

where

$$V = [v_1 \quad v_2 \quad \cdots \quad v_n],\ v_i = Q_{11}(s_i)\, f_i,\ i = 1, 2, \cdots, n, \tag{19}$$

and

$$W = [w_1 \quad w_2 \quad \cdots \quad w_n],\ w_i = Q_{21}(s_i)\, f_i,\ i = 1, 2, \cdots, n. \tag{20}$$

and $f_i \in \mathbf{C}^r$, $i = 1, 2, \cdots, n$, are a group of free parametric vectors and satisfy the following constraints:

Constraint 1: $s_i = \bar{s}_j \Leftrightarrow f_i = \overline{f_j}$, $i,\ j = 1, 2, \cdots, n$;

Constraint 2: $\det(V) \neq 0$.

4 Solution to Problem RESA

Due to the controllability of the matrix pair (A_f, B_f), we can now that the eigenvalues of the matrix A_{fc} can be assigned arbitrarily via state feedback. Thus the eigenvalues s_i, $i = 1, 2, \cdots, n$, of the matrix A_c can be assigned to those A_{fc} via state feedback. Then the relation (10) in Problem RESA is satisfied and the main task left for the solution to Problem RESA is to design a state feedback such that (11) holds.

Clearly, denote

$$V_f = [v_{f1} \quad v_{f2} \quad \cdots \quad v_{fn}], \tag{21}$$

then equations in (12) can be written into the following compact form

$$V_f \Lambda = A_f V_f + B_f K_f V_f. \tag{22}$$

Further, denote

$$W_f = K_f V_f, \tag{23}$$

then equation (22) is changed into the following form

$$V_f \Lambda = A_f V_f + B_f W_f. \tag{24}$$

Due to the controllability of the matrix pair (A_f, B_f), applying a series of element matrix transformations to matrix $[A_f - sI \ \ B_f]$, we can obtain a pair of unimodular matrices $P_f(s) \in R^{n\times n}[s]$ and $Q_f(s) \in R^{(n+m)\times(n+m)}[s]$ satisfying the following equation:

$$P_f(s)[A_f - sI \quad B_f]Q_f(s) = [0 \quad I], \ \forall s \in \mathbf{C}. \tag{25}$$

Partition $Q_f(s)$ into the following form

$$Q_f(s) = \begin{bmatrix} Q_{11}^f(s) & Q_{12}^f(s) \\ Q_{21}^f(s) & Q_{22}^f(s) \end{bmatrix}, \ Q_{11}^f(s) \in R^{n\times m}[s]. \tag{26}$$

By utilizing the same method in Lemma 1, we can obtain the following theorem, which gives solutions to equations (23) and (24).

Theorem 1. Given matrices A_f and B_f with $\text{rank}(B_f) = m$, and a group of self-conjugate distinct complex numbers s_i, $i = 1, 2, \cdots, n$, if the matrix pair (A_f, B_f) is controllable, then the parametric expressions of all the state feedback gain matrix K_f in (23) can be given by

$$K_f = W_f V_f^{-1}, \tag{27}$$

where

$$V_f = [v_{f1} \quad v_{f2} \quad \cdots \quad v_{fn}], \ v_{fi} = Q_{11}^f(s_i)g_i, \ i = 1, 2, \cdots, n, \tag{28}$$

and

$$W_f = [w_{f1} \quad w_{f2} \quad \cdots \quad w_{fn}], \ w_{fi} = Q_{21}^f(s_i)g_i, \ i = 1, 2, \cdots, n, \tag{29}$$

and $g_i \in \boldsymbol{C}^m$, $i = 1, 2, \cdots, n$, are a group of free parametric vectors and satisfy the following constraints:

Constraint 3: $s_i = \bar{s}_j \Leftrightarrow g_i = \overline{g_j}$, $i, j = 1, 2, \cdots, n$;

Constraint 4: $\det(V_f) \neq 0$.

Substituting (19) and (28) into (11), obtains

$$J_i = \left\| Q_{11}(s_i) f_i - Q_{11}^f(s_i) g_i \right\|^2, \ i = 1, 2, \cdots, n. \tag{30}$$

By using orthogonal projection, we can obtain

$$g_i = \left[\left(Q_{11}^f(s_i)\right)^H Q_{11}(s_i) \right]^{-1} \left(Q_{11}^f(s_i)\right)^H Q_{11}(s_i) f_i, \ i = 1, 2, \cdots, n, \tag{31}$$

which minimize the indexes in (11). Further, let

$$\Sigma_i = \left[\left(Q_{11}^f(s_i)\right)^H Q_{11}(s_i) \right]^{-1} \left(Q_{11}^f(s_i)\right)^H Q_{11}(s_i), \ i = 1, 2, \cdots, n, \tag{32}$$

then (31) is changed into

$$g_i = \Sigma_i f_i, \ i = 1, 2, \cdots, n. \tag{33}$$

Substituting (33) into (28) and (29), yields

$$v_{fi} = Q_{11}^f(s_i)\Sigma_i f_i, \ i = 1, 2, \cdots, n, \tag{34}$$

and

$$w_{fi} = Q_{21}^f(s_i)\Sigma_i f_i, \ i = 1, 2, \cdots, n. \tag{35}$$

Thus, the redesigned state feedback gain can be given by

$$K_f = W_f V_f^{-1}, \tag{36}$$

where

$$V_f = [v_{f1} \quad v_{f2} \quad \cdots \quad v_{fn}],\ v_{fi} = Q_{11}^f(s_i)\Sigma_i f_i, \tag{37}$$

and

$$W_f = [w_{f1} \quad w_{f2} \quad \cdots \quad w_{fn}],\ w_{fi} = Q_{21}^f(s_i)\Sigma_i f_i. \tag{38}$$

In order to guarantee the realness of the gain matrix K_f in (36), the following constraint must hold:

Constraint C1: $s_i = \bar{s}_j \Leftrightarrow \Sigma_i = \overline{\Sigma_j}$, $f_i = \overline{f_j}$, $i, j = 1, 2, \cdots, n$;

Moreover, Constraints 2 and 4 must hold, and are clearly equivalent with the following two constraints, respectively,

Constraint C2: $\det[Q_{11}(s_1) f_1 \quad Q_{11}(s_2) f_2 \quad \cdots \quad Q_{11}(s_n) f_n] \neq 0$;

Constraint C3: $\det[Q_{11}^f(s_1)\Sigma_1 f_1 \quad Q_{11}^f(s_2)\Sigma_2 f_2 \quad \cdots \quad Q_{11}^f(s_n)\Sigma_n f_n] \neq 0$.

From the above reductions, we can give the following theorem, which gives the solution to Problem RESA.

Theorem 2. Given the controllable normal system (1) and the controllable fault system (3), and a group of distinct and self-conjugate scalars s_i, $i = 1, 2, \cdots, n$. Then all the desired solutions K_f in Problem RESA can be given by (36) with the parametric vectors $f_i \in \boldsymbol{C}^r$, $i = 1, 2, \cdots, n$, satisfying Constraints C1-C3.

According to Theorem 2 and the above deductions, the following algorithm for Problem RESA can be proposed as follows.

Algorithm RESA:

1. Calculate a pair of unimodular matrices $P(s)$ and $Q(s)$ satisfying (16), and partition $Q(s)$ as in (17);
2. Calculate a pair of unimodular matrices $P_f(s)$ and $Q_f(s)$ satisfying (25), and partition $Q_f(s)$ as in (26);
3. Find a group of parameters $f_i \in \boldsymbol{C}^r$, $i = 1, 2, \cdots, n$, satisfying Constraints C1-C3, and calculate the matrices W_f and V_f according to (38) and (37), respectively;
4. Calculate the state feedback gain matrix K_f according to (36).

5 An Illustrative Example

Consider a normal linear system and its corresponding fault linear system with the following parameters:

$$A=\begin{bmatrix}0&1&0\\0&0&1\\0&0&1\end{bmatrix},\quad B=\begin{bmatrix}0&0\\0&1\\1&0\end{bmatrix},\quad A_f=\begin{bmatrix}0&1&0\\0&0&1\\0&0&-1\end{bmatrix},\quad B_f=\begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}.$$

Easily, we can find that the matrix pair (A,B) and (A_f, B_f) are both controllable. In this example, we choose the eigenvalues of the normal closed-loop system as $s_1=\overline{s_2}=-2\pm 3i$, $s_3=-4$. Algorithm RESA is utilized to solve this reconfiguration problem. The results of each step are given as follows.

1) Obtain the following matrices satisfying (16) as

$$P(s)=I,\ Q(s)=\left[\begin{array}{cc|ccc}1&0&1&-1&0\\s&0&1+s&-s&0\\1&1&0&1&1\\\hline s-1&s-1&0&s-1&s\\s^2-1&-1&s(s+1)&-s^2&-1\end{array}\right].$$

2) Obtain the following matrices satisfying (25) as

$$P_f(s)=I,\ Q_f(s)=\left[\begin{array}{cc|ccc}1&0&0&0&0\\s&0&1&0&0\\0&1&0&0&0\\\hline s^2&-1&s&1&0\\0&1+s&0&0&1\end{array}\right].$$

3) -4) Denoting $f_i=\begin{bmatrix}a_i\\b_i\end{bmatrix}$, $i=1,2,3,4$. Then from (37) and (38), we can get the parametric expressions of V_f and W_f. Thus we can get the parametric expression of the redesigned state feedback gain matrix K_f from (36).

Specially, choosing a group of the parametric vectors as

$$f_1=\overline{f_2}=\begin{bmatrix}3-2i\\-4+7i\end{bmatrix},\quad f_3=\begin{bmatrix}-2\\5\end{bmatrix},$$

then we can calculate that

$$V_f=\begin{bmatrix}3-2i&3+2i&-2\\13i&-13i&8\\-4+7i&-4-7i&5\end{bmatrix},\quad W_f=\begin{bmatrix}-35-33i&-35+33i&-37\\-17-19i&-17+19i&-15\end{bmatrix},$$

and

$$K_f=\begin{bmatrix}91&-30&77\\25&-10&23\end{bmatrix}.$$

Moreover, from (18) we obtain

$$K=\begin{bmatrix}3&-9&21\\13&-30&77\end{bmatrix}.$$

For convenience, we call the closed-loop systems of the normal system under K as system 1, the closed-loop system of the fault system under K_f as system 2 and the closed-loop system of the fault system under K as system 3, respectively. The errors between the outputs of system 1 and system 2 are given in Fig.1 and Fig.2, respectively and the simulation results show that the redesigned feedback K_f is effective. Moreover, the eigenvalues of system 3 are 67.4133, -0.2066+0.9042i, and -0.2066-0.9042i, in which 67.4133 is an unstable eigenvalue, while the eigenvalues of system 1 are the same with those of system 2.

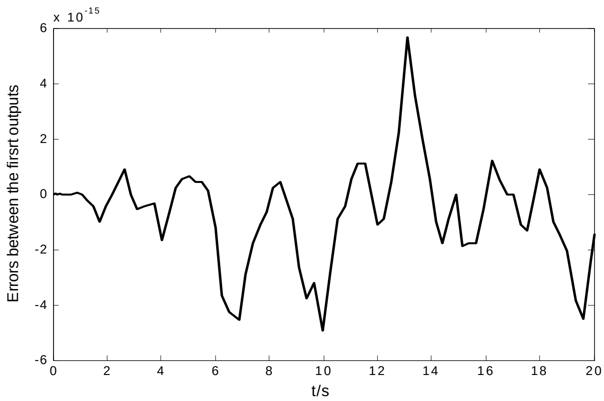

Fig. 1. Errors between the first outputs of system 1 and system 2

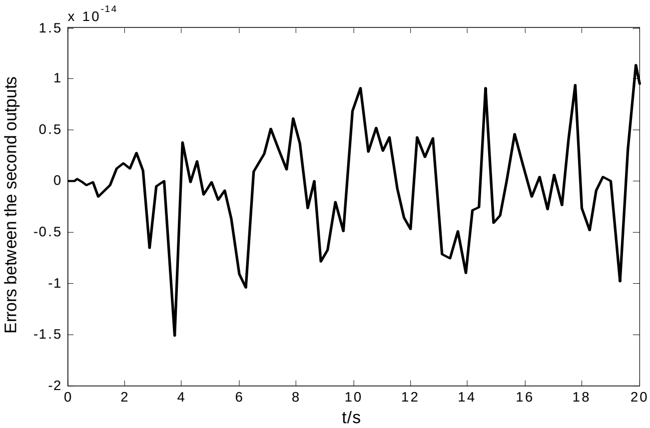

Fig. 2. Errors between the second outputs of system 1 and system 2

6 Conclusions

In this paper reconfiguring linear control systems via state feedback eigenstructure assignment is investigated. By utilizing the freedom degrees offered by a parametric result of eigenstructure assignment in linear control systems, a parametric expression for all the state feedback gain matrices, which can recover the eigenvalues of the normal closed-loop system and make the eigenvectors of the fault closed-loop system as close to those of the normal closed-loop system as possible, is established and an algorithm for this design is proposed. The parametric method offers all the design degrees of freedom, which can be further utilized to satisfy certain specifications in control system designs, such as robustness etc. An illustrative example and the simulation figures show the effect of the proposed algorithm.

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