Depth Resolution Improvement for OCT Signals Based on Super-resolution Algorithm

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Abstract

In this paper, a superresolution algorithm using the minimum entropy criterion is presented to improve the depth resolution of Optical Coherent Tomography (OCT) signals. The proposed method, which is an improved minimum entropy deconvolution (MED) technique, exploits the second order statistics(SOS) of the coherence function as an extra constraint in the procedure of entropy minimization. We denote the proposed method as IMED. The classical MED performs the deconvolution by maximizing an entropy norm with respect to the coefficients of a linear operator. In comparison, IMED not only performs the deconvolution, but also recovers the missing frequencies by maximizing the norm in respect to them using a nonlinear iterative algorithm.

Keyword: OCT(Optical Coherent Tomography); Superresolution; Deconvolution.

I. Introduction

OCT is an emerging imaging technology with applications in biology, medicine, and materials investigations. Attractive features include high cellular-level resolution, real-time acquisition rates, and spectroscopic feature extraction in a compact noninvasive instrument. OCT has been applied to a wide range of biological, medical, and materials investigations. To improve the depth resolution of OCT signals, the spectral width needed to be expanded by hardware or numerical methods[1-6]. There are two major numerical methods, spectral shaping methods[2-4] and deconvolution techniques[4-6].

The minimum entropy deconvolution, which is developed primarily for seismic signals deconvolution[7], has been developed for blind equalization in wireless communication systems, image deconvolution and period estimation et al.. It is seen that a large existing higher order statistics (HOS) based blind equalization algorithms are directly related to the scale-invariant cost function used in the MED [8]. Higher order statistics (HOS) have been applied successfully to the problem of blind deconvolution, mainly because of their ability to preserve the true system phase, and their robustness to additive Gaussian noise of unknown covariance [9]. Since the MED operator is linear, it can not recover the missing frequencies. An improved MED method with frequency-domain constraints (FMED)[10] is proposed to recover the missing frequencies.

It is seen that HOS estimator has higher variance and SOS estimator has lower one[9]. In its applications in seismic signal processing, the classical MED is sensitive to the length of the inverse filter, strong reflections in the reflectivity sequence and to what extent the sparsity assumption of the reflectivity sequence is satisfied [7]. To overcome the above limitations of the classical MED, by supposing that the SOS of the coherence function can be obtained from the OCT signals, the proposed IMED exploits the SOS of the coherence function as an extra constraint in the procedure of entropy minimization. Furthermore, IMED adopts the same algorithm used by FMED to recover the missing frequencies. Applications of our method on experimental OCT signals show that promising results can be obtained.

II. Method

The OCT signal can be modeled as a linear shift invariant system(LSI)[5,6]. In our problem, the main objective is to reconstruct the reflectivity sequence $s(t)$, which is distorted by a band-limited LSI system (coherence function), from the received OCT signal $x(t)$, which can be represented as:

$$x(t) = b(t) * s(t) + n(t) \quad (1)$$

where $b(t)$ is the impulse response of the system, $*$ denotes the convolutional product, and $n(t)$ is the additive Gaussian noise.

To obtain an estimation of the reflectivity sequence $\hat{s}(t)$, the classical MED optimizes an inverse filter (IF) $f(t)$ by maximizing the scale-invariant, variable norm cost function

$$V(\hat{s}(t)) = \frac{\sum_{t=1}^{N} \hat{s}^4(t)}{\left(\sum_{t=1}^{N} \hat{s}^2(t)\right)^2} \quad (2)$$

where N is the sample number of the reflectivity sequence, and

$$\hat{s}(t) = f(t) * x(t). \quad (3)$$

It is well known that the measure defined in Eq. (2) is the normalized kurtosis[8].By setting a fixed length of the IF $f(t)$ as L, the classical MED algorithm written in matrix notation is expressed as

$$Rf = g(f), \quad (4)$$

where R is the Toeplitz matrix of the OCT signal $x(t)$ with size $L \times L$ and the vector $g(f)$ is the cross-correlation between vectors $c(t) = \hat{s}^3(t)$ and $x(t)$. Eq. (4) must be solved through an iterative algorithm:

$$f^{(n)} = R^{-1} g(f^{(n-1)}), \quad (5)$$

where the upper index n denotes iteration number. In each iteration, the system is solved with Levinson's algorithm. The initial value of the IF is $f^{(0)} = (0,0,0,...,1,...,0,0,0)$.

To overcome the above limitations of the classical MED, by supposing that the SOS of the coherence function can be obtained from the OCT signal, the proposed IMED exploits the SOS of the coherence function as an extra constraint in the procedure of entropy minimization. Furthermore, IMED adopts the same algorithm used by FMED to recover the missing frequencies. It is known that the auto-correlation of the coherence function $b(i)$ can be estimated by windowing the auto-correlation of the OCT signal $x(i)$:

$$m_2^b(j) = m_2^x(j) d(j), \quad (6)$$

where $d(j)$ is a normal 1D window (e.g., Gaussian, boxcar) function centered at zero time, $-q < j < q$, where q is the coherence function length, and the auto-correlation of the OCT signal $x(i)$ is defined:

$$m_2^x(j) = \sum_i x(i)x(i+j) \,. \tag{7}$$

We can estimate the amplitude spectrum $|B(\omega)|$ of the coherence function using the auto-correlation obtained by Eq. (6):

$$|B(\omega)| = \left|FFT\left\{m_2^b(j)\right\}\right|^{1/2} , \tag{8}$$

where $FFT\{\ \}$ is 1D Fourier transform.

We will assume that the frequency range of the coherence function $b(t)$ is $[\omega_L, \omega_H]$, and its amplitude spectrum $|B(\omega)|$ in the frequency range $[\omega_L, \omega_H]$ is known. In each iteration, we replace the amplitude spectrum of IF obtained by (5) as the reciprocal of $|B(\omega)|$, in other words, the maximization of the entropy norm is subjected to the following constraint:

$$|F(\omega)| = \frac{1}{|B(\omega)|} \qquad \omega \in \left\{[\omega_L, \omega_H], |B(\omega)| > 0\right\} . \tag{9}$$

To recover the frequencies outside $[\omega_L, \omega_H]$, we adopt the nonlinear algorithm using by FMED [10]. Denoting the frequency spectrum of the reflectivity sequence obtained by Eq. (3) as $\hat{S}(\omega)$, and that of the vector $c(t) = \hat{s}^3(t)$ as $C(\omega)$, we obtain the estimation of the reflectivity sequence $\hat{s}'(t)$ with frequencies outside $[\omega_L, \omega_H]$ in the following way:

$$\hat{S}'(\omega) = \begin{cases} \hat{S}(\omega) & \omega \in [\omega_L, \omega_H] \\ \alpha C(\omega) & \omega \notin [\omega_L, \omega_H] \end{cases} , \tag{10}$$

where α is an energy adjusting factor to make sure that the amplitude spectrum of reflectivity sequence is consistent before and after the spectrum extending:

$$\alpha = \frac{\sum_{\omega=\omega_L}^{\omega_H} \left\|\hat{S}(\omega)\right\|_2}{\sum_{\omega=\omega_L}^{\omega_H} \left\|C(\omega)\right\|_2} \,. \tag{11}$$

The whole flowchart of IMED is as follows:

1. The algorithm is initialized by letting $f^{(0)} = (0,0,0,\ldots,1,\ldots,0,0,0)$.
2. Inverse filter $F(\omega)$ is computed by Eq. (5).
3. Amplitude spectrum of inverse filter $F(\omega)$ is updated by Eq. (9) while its phase spectrum is remained.
4. The estimated the reflectivity sequence $\hat{s}(t)$ is computed by Eq. (3).
5. Extend frequency range of the estimated reflectivity sequence $\hat{s}'(t)$ by Eq. (10), and replace $\hat{s}(t)$ as $\hat{s}'(t)$.

Convergence is checked by pre-setting conditions and a new iteration starts in step 2.

III. Results

Computer simulations are conducted to illustrate the performance of the proposed approach. We evaluate the sparsity of the input sequence by the ratio between the number of zeros N_Z and the length of the whole sequence N :

$$S = \frac{N_Z}{N}. \quad (12)$$

An example with synthetic signal is shown in figure 1. Figure 1a shows the impulse response of the channel. Figure 1b and 1c show the first 200 samples of input sequence with $S = 0.4$ and the corresponding output sequence respectively. The length of the output sequence is 600 samples. Figure 1d and 1e show the first 200 samples of

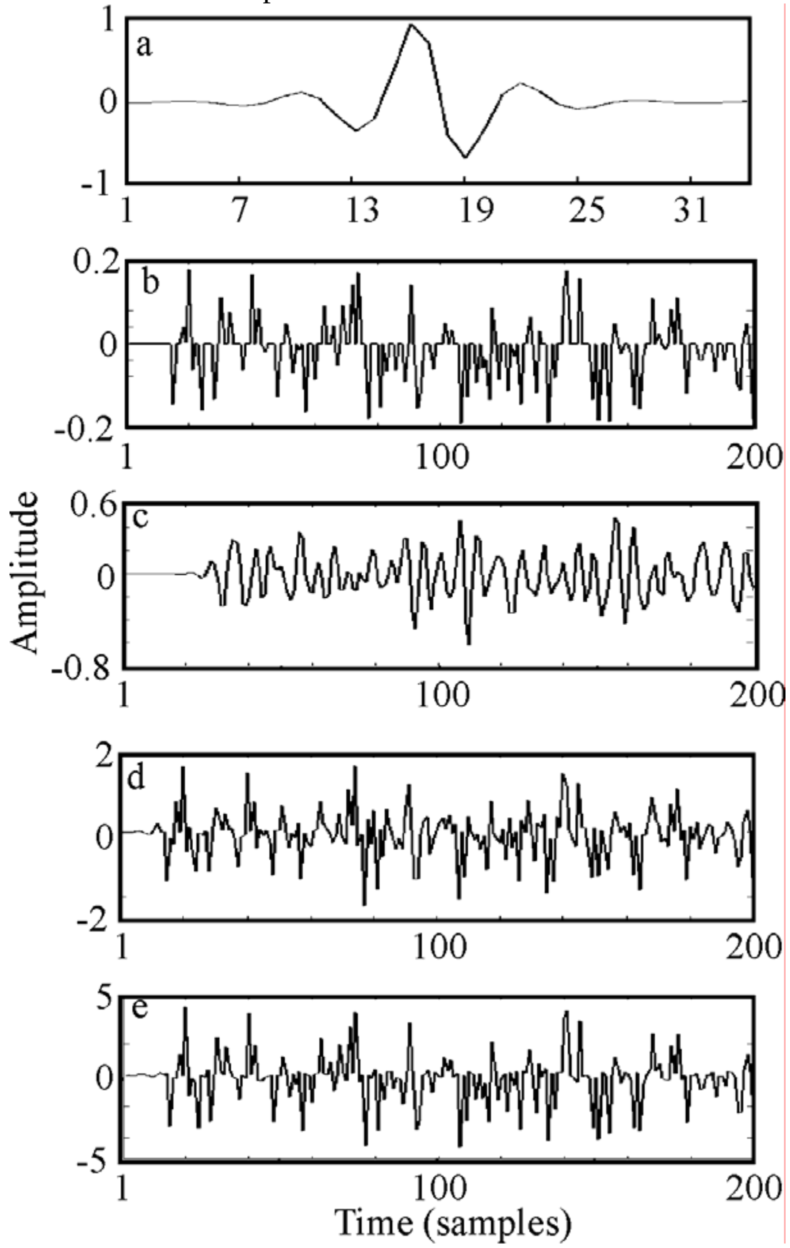

Fig. 1. Simulation for noiseless signal case. (a) Impulse response of system. (b) Input sequence of system. (c) Output sequence of system. (d) Estimation of the input sequence by MED. (e) Estimation of the input sequence by IMED

the estimated input sequences by MED and IMED after 10 iterations respectively. The frequency range used in IMED in this experiment is $[14Hz, 67Hz]$. It is seen that IMED can obtain better result than does MED.

To illustrate the behavior of the algorithm under noisy conditions, Gaussian noise has been added to the synthetic signal shown in figure1 with a signal-to-noise ratio of 25dB. The noisy signal is shown in figure 2a. Figure 2b and 2c show the first 200 samples of the results obtained by MED and IMED respectively.

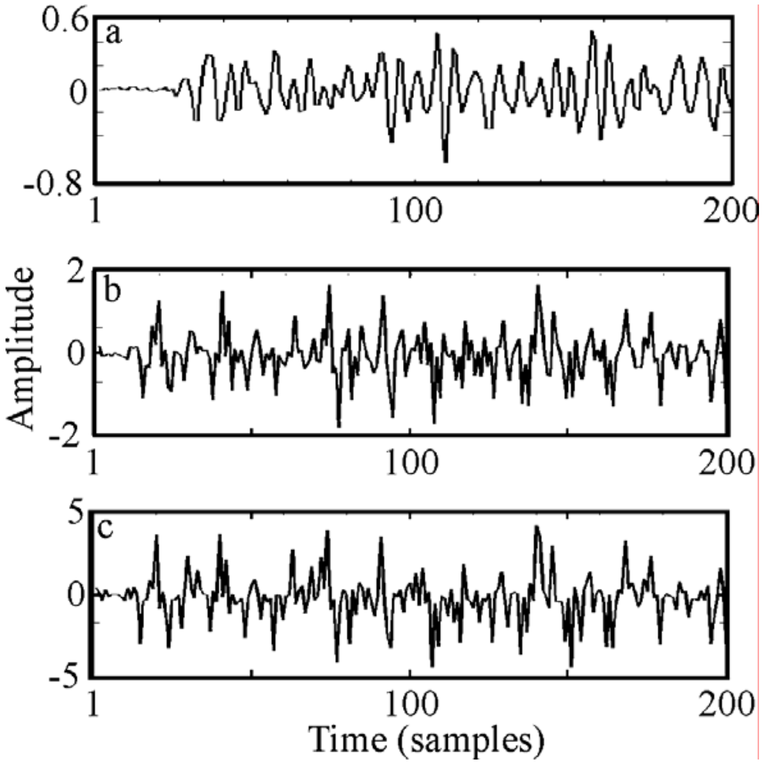

Fig.2. Simulation for noisy signal case. (a) Output sequence of system with 25dB additive noise. (b) Estimation of the input sequence by MED. (c) Estimation of the input sequence by IMED

The original OCT image shown in Fig. 3a is used to evaluate IMED. Fig. 3b shows the result after resolution improvement by our method. The sections marked by rectangle in Fig. 3a and 3b are depicted in Fig. 3c and 3d respectively. It is seen that the resulted OCT images obtained by our method give more details because of the resolution improvement.

In Fig. 4, we evaluate our method using a OCT profile. OCT signals before (left) and after (right) resolution improvement are given in the first row, and two zoomed segments are shown in the second row. The envelopes of the OCT signals shown in the first two rows are shown in the third and fourth rows respectively. It is seen that the waveforms in the resulted OCT signals (envelope) are becoming narrow and sharp. In the fourth row, we frame the waveforms corresponding to a reflective surface before (left) and after (right) resolution improvement by two green rectangles. And the width of the left rectangle is about 2.1 times of that of the right one. In the bottom row, the amplitude spectra of the OCT signals shown in the first row are given. As we expected, IMED recovers OCT signals with some frequencies that are missed in the original OCT signals.

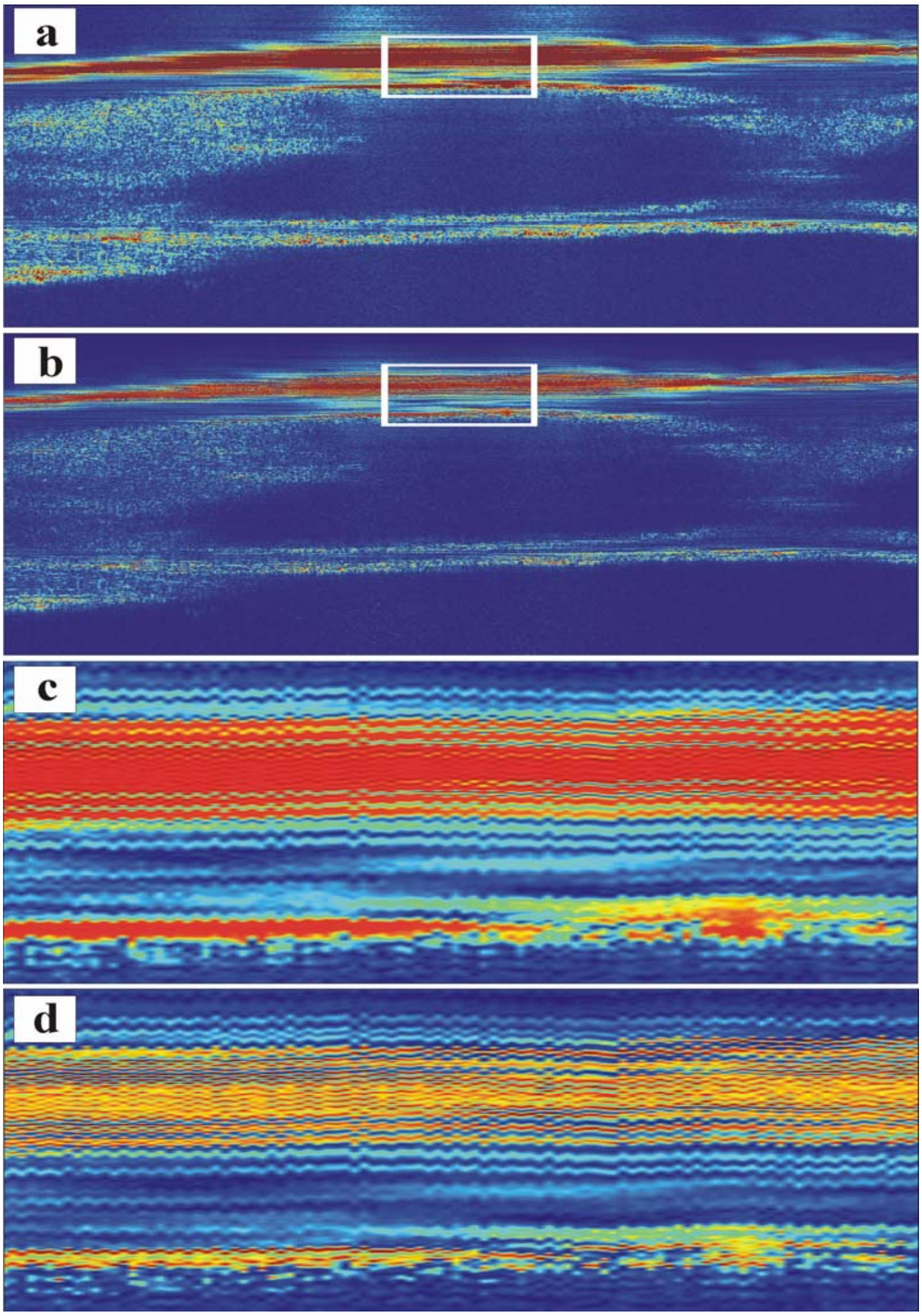

Fig. 3. OCT images (a) before and (b) after resolution improvement, (c) and (d) are the zoomed sections of areas marked by the rectangles in (a) and (b) respectively

Ⅳ. Conclusion

This paper presented a superresolution technique, IMED, to improve the depth resolution of OCT signals. The proposed method, which is an improved minimum entropy deconvolution technique, exploits the second order statistics of the coherence function as an extra constraint in the procedure of entropy minimization. In comparison with MED, IMED not only performs the deconvolution, but also recovers the missing frequencies by maximizing the norm in respect to them using a nonlinear iterative algorithm.

We have applied IMED on synthetic signals based on convolution model and experimental OCT signals. It is seen that IMED outperforms MED in both situations. In the case of experimental OCT signals, the envelope width of the coherence function after resolution improvement is about 0.48

times of that of the original one, and thus the OCT depth resolution is improved by a factor of 2.1 in our experiments.
In this work, we assume that the coherence function is invariant, to extend this work for variant coherence function case is under way.

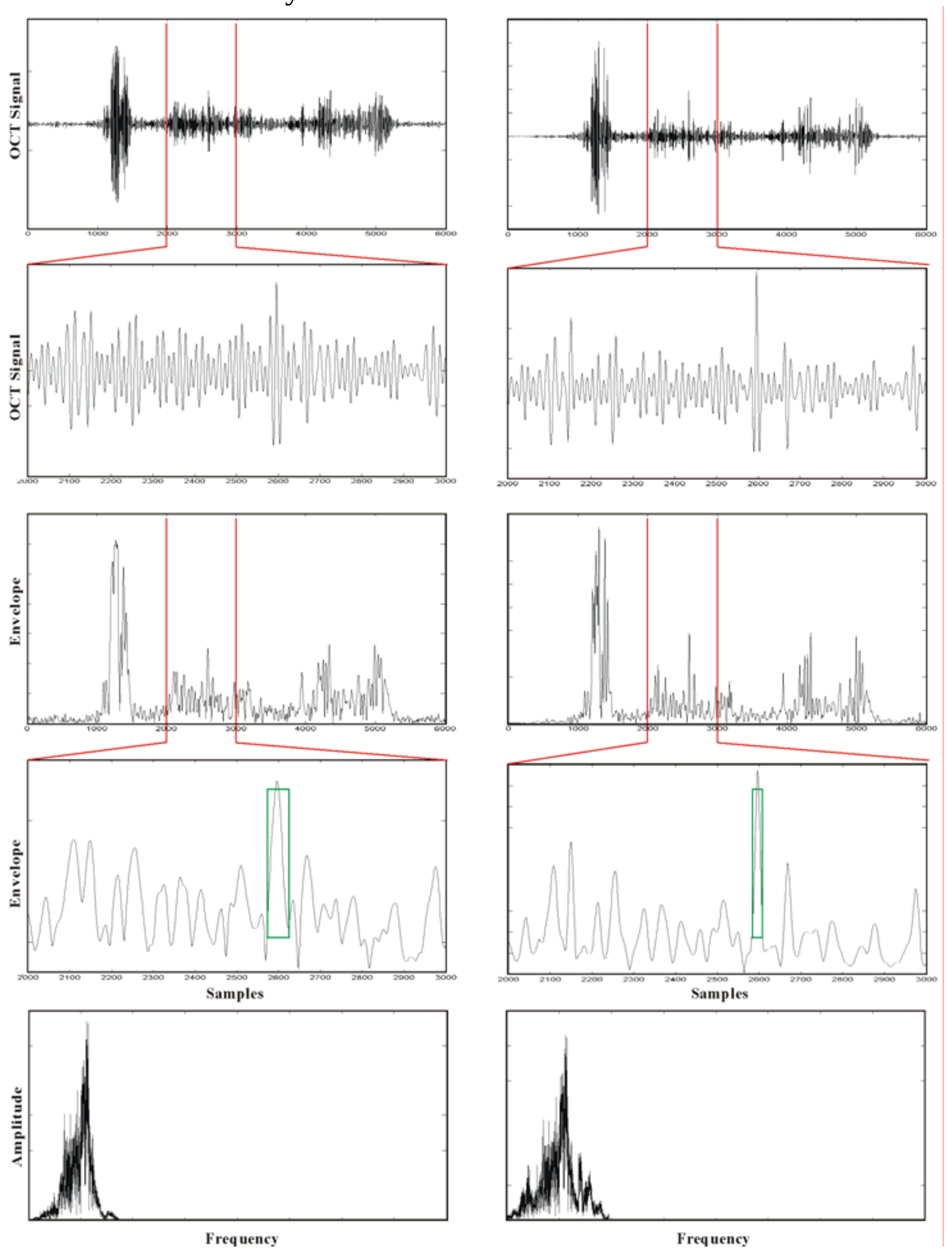

Fig. 4. Top two rows: the OCT signals before (left) and after (right) resolution improvement. The 3th and 4th rows: the envelopes of the OCT signals shown in the first two rows. The bottom row: the amplitude spectra of the OCT signals shown in the first rows

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