Estimation of the Broadening Errors in Doppler Spectra Computed Using Wavelet Transform

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Abstract

Wavelet transform (WT), which has a flexible time-frequency window, is particularly suitable for computing the time frequency distribution of nonstationary signals. In recently years, the WT has been used to investigate its advantages and limitations for the analysis of Doppler blood flow signals. In these studies, the estimated spectral width of Doppler blood flow signals using the WT might include significant window and nonstationarity broadening errors. These broadening errors of the time-varying spectrum were clearly undesirable since it would tend to mask the effect of flow disturbance on the spectra width. In this paper, a method based on the mean root-mean square-error (RMS) spectral width has been proposed to estimate the broadening errors in the WT spectrum. The results show that using this method, the increases in the RMS spectral width can be calculated and the spectral width estimation based on the WT can be corrected.

Keyword: Wavelet transform; Doppler blood flow, Spectral broadening

I. Introduction

The Doppler ultrasound is widely used to diagnose arterial occlusive diseases. Diagnostic information is extracted from the Doppler blood flow signal resulting from the backscattering of the ultrasound beam by moving red blood cells [1,2,3]. The accuracy of the diagnosis is dependent on physical properties of the pulsed ultrasonic Doppler blood flow detector and the Doppler signal analysis technique. Therefore, the analysis algorithm used to process the Doppler signal should have high accuracy for the estimation of the frequency waveform and high sensitivity for the detection of turbulence –induced spectral broadening.

Flow disturbance caused increase in the Doppler spectral width is used to detect atherosclerotic lesions in arteries. The width of the Doppler spectrum influences the sensitivity of the flow disturbance detection and the mean velocity estimation [1,2]. The resolution of the spectral estimator used limits the detection sensitivity of disturbance-induced spectral broadening. Conventional spectral analysis of the Doppler signal is performed using the STFT [1,2,3], in which the signal is divided into small sequential or overlapping segments and the fast Fourier transform is applied to each one. There is a tradeoff when using the STFT between the distortion and poor spectral resolution introduced by short data windows and the spectral broadening that arises from nonstationary characteristics of the signal when using longer data windows. Much progress has been made in identifying and quantifying spectral broadening resulting from blood flow nonstationarity [10,11,12,13].

On the contrary to the STFT, the WT incorporates the concept of scale into the transform, which gives better time-frequency resolution: a compressed wavelet for analyzing high frequency details and a dilated wavelet for detecting lower frequency underlying trend. This method is particularly suitable for analyzing signals having a relatively wide bandwidth and rapidly changing with time [4,5].

Many people have used the WT for analyzing Doppler blood flow signals[6,7,8,9]. The results showed that the WT can not only be used as an alternative signal processing tool to the STFT for Doppler blood flow signals, but also can generate a time-frequency representation with better resolution than the STFT. In addition, the WT method can provide both satisfactory mean frequency and maximum frequency waveforms. In these studies, the estimated spectral width of Doppler blood signals using the WT may include significant widow and nonstationarity broadening errors. These broadening errors of the time-varying spectrum were clearly undesirable since it would tend to mask the effect of flow disturbance on the spectral width.

In present study, a method based on the mean RMS spectral width has been proposed to estimate the broadening errors in the WT spectrum. The increases in the RMS spectral width can be calculated and the spectral width estimation based on the WT can be corrected. In following sections, we first briefly describe the theoretical analysis of window and nonstationarity broadening when using the modified Morlet wavelet to estimate Doppler blood flow spectrum. The spectral width correction method based on the mean RMS spectral width is demonstrated. Then the simulation experiment is described, followed by results, discussions and conclusions.

II. Methods

A. Doppler Spectral Width

Several measures of the spectral width were used in the Doppler spectrum analysis, such as: the -3 dB frequency, the RMS width, the SBI, etc. In this paper, we investigate the RMS spectral width at time t, $\sigma(t)$, defined by

$$\sigma(t) = \left[\frac{\int [f_m(t) - f]^2 S(f,t) df}{\int S(f,t) df} \right]^{1/2} \qquad (1)$$

where $f_m(t)$ is the mean frequency waveform of the Doppler blood flow's time-frequency representation $S(f,t)$. This measure potentially allows the correction for spectral broadening [10,12,13,15]. The continuous wavelet transform (CWT) can be expressed in a time-frequency version as [6]:

$$CWT\ (f) = \int_{-T/2}^{T/2} x(t) \sqrt{f / f_0}\, \varphi\left(\frac{t}{f_0 / f}\right) dt \qquad (2)$$

with the wavelet duration T, the basic frequency f_0 and the analyzing frequency f .The analysis functions $\sqrt{f / f_0}\,\varphi(\frac{t}{f_0 / f})$,called wavelets, are scaled and shifted versions of the assumed prototype function $\varphi(t)$, the mother wavelet.

The modified Morlet wavelet ($\sigma = 5$) which has a good compromise between time and frequency resolution is defined as [6,18,19]:

$$\varphi_{\sigma_t, f_0}(t) = e^{-t^2 / 2\sigma_t^2} e^{-j2\pi f_0 t} \qquad (3)$$

where σ_t determines the duration of the Gaussian window, f_0 is the basic frequency of the wavelet. CWT with modified Morlet form (2) (3) is:

$$CWT(f) = \int_{-T/2}^{T/2} x(t)\sqrt{f/f_0}\, e^{-t^2/2\sigma_t^2 (f_0/f)^2} e^{-j2\pi ft} dt \quad (4)$$

where $f_0 / f (f_0 \geq f)$ could be regarded as the scale s in the WT, then:

$$CWT_s(f) = \int_{-T/2}^{T/2} x(t) \frac{1}{\sqrt{s}} e^{-t^2/2\sigma_t^2 s^2} e^{-j2\pi ft} dt \quad (5)$$

The CWT could be considered as a special version of the STFT with the window:

$$w_s(t) = \frac{1}{\sqrt{s}} e^{-t^2/2\sigma_t^2 s^2} \quad (6)$$

this window $w_s(t)$, whose duration is determined not only by the σ_t but also by the scale indicator s, is flexible.

Several models of the process of the Doppler signal generation have been published [16,17]. Due to the pulsatile nature of the blood flow, the Doppler signal $x(t)$ windowed by $w(t)$ is not stationary, it can be written as [10]:

$$x_{wx}(t) = w^1(t) x(t) \quad (7)$$

where

$$w^1(t) = e^{j\theta(t)} w(t) \quad (8)$$

and $e^{j\theta(t)}$ is a frequency-shifting shift describing the variation in the spectral mean frequency waveform $f_m(t)$. When the frequency of frequency-shifting term $e^{j\theta(t)}$ varies linearly with time, we can consider that the duration and the rate of change of the mean frequency are sufficiently small that only the second order term needs to be considered. The analysis will not be valid for long-duration windows at the turning points of the velocity waveform. In this case:

$$\theta(t) = \pi\beta t^2 \quad (9)$$

where β is the slope of the frequency variation of the mean frequency waveform $f_m(t)$. The expected energy spectrum of $x_{wx}(t)$ is:

$$\varepsilon[S_{wx}(f,t)] = |W^1(f,t)|^2 * S_x(f,t) \quad (10)$$

where $|W^1(f,t)|^2$ is the energy spectrum of $w^1(t)$. $S_x(f,t)$ is the spectrum of Doppler signal $x(t)$. For the spectral analysis using the WT, the window $w_s(t)$ in (6) will be used as $w(t)$ in (8). In this case, the energy spectrum of $w^1(t)$ is:

$$|W_s^1(f,t)|^2 = \frac{\sigma_t}{\sqrt{2}\sigma_{rms}} e^{-\frac{f^2}{2\sigma_{rms}^2}} \quad (11)$$

where the RMS width of $|W_s^1(f,t)|^2$ is:

$$\sigma_{rms}(s,t) = \sqrt{\frac{1}{8\pi^2\sigma_t^2 s^2} + \frac{\sigma_t^2 s^2 \beta^2}{2}} \quad (12)$$

B. *Broadening Correction*

Because a signal has many different frequency components, so does the scale s using the WT. To evaluate the root mean square width of the broadening using the WT, we can obtain the mean RMS width of $|W_s^1(f,t)|^2$ in (11):

$$\sigma_{f_w}^2(t) = \frac{\int \sigma_{rms}(s,t)S(s,t)ds}{\int S(s,t)ds} \quad (13)$$

where $S(s,t)$ is the estimated time-frequency representation using the WT. The RMS spectral width of $S_x(f,t)$ can be obtained form (10) as:

$$\sigma_{f_x}^2(t) = \sigma_{f_{wx}}^2(t) - \sigma_{f_w}^2(t) \quad (14)$$

where $\sigma_{f_{wx}}^2(t)$, $\sigma_{f_w}^2(t)$ and $\sigma_{f_x}^2(t)$ are the RMS spectral widths of $x_{wx}(t)$, $w^1(t)$ and $x(t)$. $\sigma_{f_x}^2(t)$ is the true RMS spectral width of the Doppler blood flow signal.

III Simulation Experiment

To examine the method described above, we use simulated carotid Doppler signals having spectrum center-frequency and width variation similar to those found in a typical clinical study [14]. The $f_m(t)$ is shown in Fig.1.

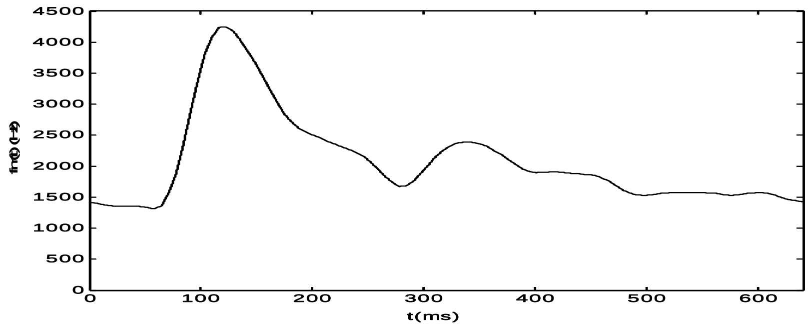

Fig. 1. Simulated mean frequency waveform $f_m(t)$

Two types of cases are considered: One has a constant RMS width of 100 Hz, the other incorporates the spectral width broadening during the decelerative phase of systole similar to that seen during the lesion-induced turbulent flow and the corresponding band width has the form:

$$\sigma_f(t) = a + b\exp[-\frac{(t-c)^2}{2d^2}] \quad (15)$$

where a=100 Hz, b=200 Hz, c=132.5 ms, and d=22.5 ms as shown in Fig.2.

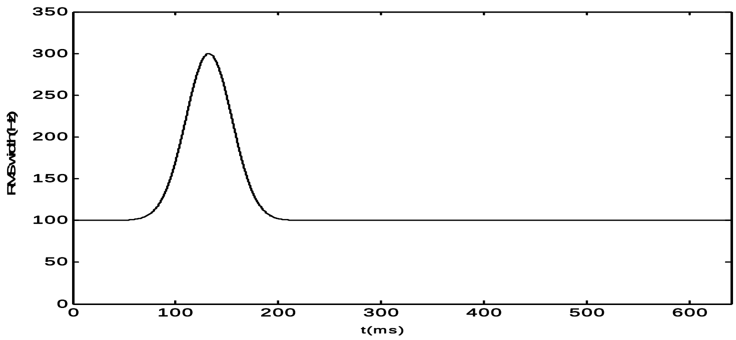

Fig. 2. A time varying spectral width waveform.

The signal power is proportional to the spectral width $\sigma_f(t)$, which simulates the increase in power along with the spectral width during the turbulent flow. In all cases the sample frequency f_s is 12.8 kHz, the cardiac cycle period T is 640ms. The power spectra of the simulated Doppler signals are estimated using the modified Morlet ($\sigma = 5$) wavelet with windows of time durations 5 ms and 10 ms. The mean frequency waveform and the RMS spectral width calculated from the spectra are averaged over 20 cardiac cycles, and the corrected estimation of the spectral width is calculated. The corrected RMS spectral widths are then compared with the theoretical one.

IV Results and Discussion

Fig.3 shows the ensemble averaged RMS spectral width waveforms of 20 cardiac cycles following the analysis using the modified Morlet WT. The RMS spectral width $\sigma^2_{f_{wx}}(t)$ waveforms, which include the effects of nonstationarity broadening and window broadening, are shown in Fig.3 (a, c, e, g). The corrected RMS spectral width waveforms $\sigma^2_{f_x}(t)$ are shown in Fig.3 (b, d, f, h). Fig.3 (a-b, e-f) show the RMS spectral width waveforms for the constant RMS spectral width of 100 Hz. Fig.3 (c-d, g-h) show the RMS spectral width waveforms for varying RMS spectral width corresponding to the time varying spectral width waveform shown in the Fig.2. Fig.3 (a-d) and Fig.3 (e-h) show the RMS spectral width waveforms following the WT analysis with 5 ms window duration and 10 ms window duration respectively. From the Fig.3 (a, c, e, g), it can be found that the overestimated spectral broadening using the WT is so large that it could completely mask the effect of flow disturbance on the spectra width. Comparing with Fig.1, we can find that the spectral broadening is obvious when the mean frequency waveform varies rapidly. This may mostly ascribe

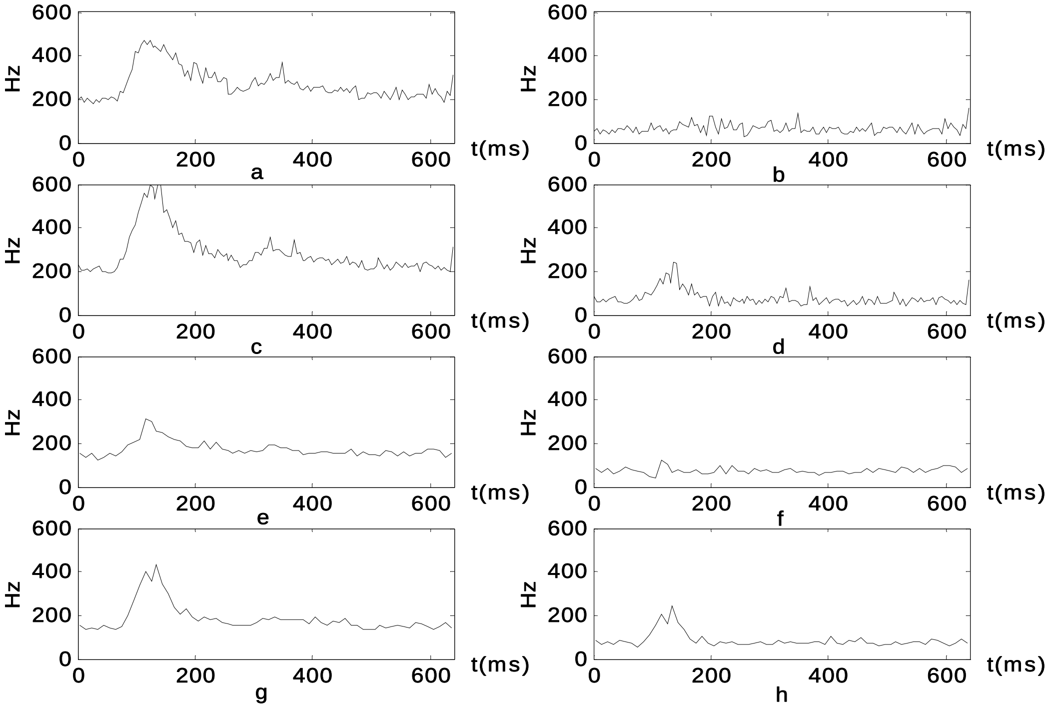

Fig. 3. The $\sigma^2_{f_{wx}}(t)$ (a, c, e, g), $\sigma^2_{f_x}(t)$ (b, d, f, h) for constant RMS spectral width of 100 Hz (a-b, e-f) and varying RMS spectral width (c-d, g-h). (a-d): 5 ms window; (e-h):10 ms window.

to the stationary assumption, which determines the nonstationarity broadening, is not valid. The overestimated spectral broadening during the mean frequency rapid changing period for varying RMS spectral width shown in Fig.3 (c, g) is less than that for the constant RMS spectral width of 100 Hz shown in Fig.3 (a, e). However overestimated spectral broadening during the mean frequency slow changing period for varying RMS spectral width shown in Fig.3 (c, g) is almost the same as that for the constant RMS spectral width of 100 Hz shown in Fig.3 (a, e). The overestimated spectral broadening extracted from the spectrum using the 5 ms window duration shown in Fig.3 (a, c) is larger than that using the 10 ms window duration shown in Fig.3 (e, g), especially when the mean frequency is changing rapidly, this may due to the fact that shorter analyzing window will lead to greater window broadening during the frequency rapid changing period. From Fig.3 (e-h), it can be found that fewer frequency fluctuations exist in the RMS spectral broadening waveform extracted from the spectrum using the WT with the 10 ms window duration. Both the window induced spectral broadening and the nonstationarity broadening are corrected in all cases. It can also be found that the corrected spectral broadening width waveforms for the constant RMS spectral width of 100 Hz and for varying RMS spectral width closely match the theoretical ones respectively.

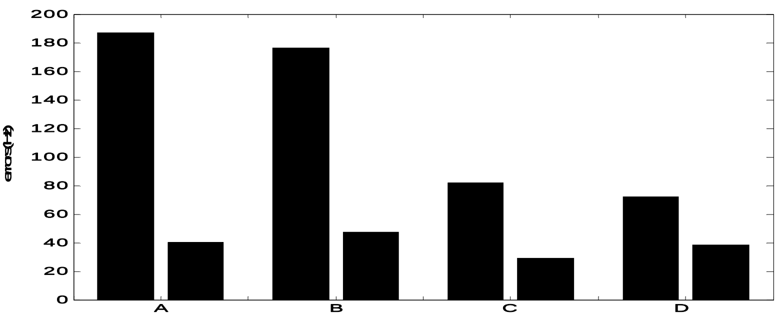

Fig. 4. The RMS errors of $\sigma^2_{f_{wx}}(t)$, $\sigma^2_{f_x}(t)$, from left to right in each group. A, C: signals with a constant RMS width; B, D: signals with a varying RMS width; A, B: 5 ms window; C, D: 10 ms window.

In Fig. 4, from left to right in each group, the normalized root-mean-square errors of the RMS spectral width waveforms $\sigma^2_{f_{wx}}(t)$ and the corrected RMS spectral width waveforms $\sigma^2_{f_x}(t)$ are shown. The normalized root-mean-square errors of the RMS spectral width waveforms are calculated from ensemble average of 20 realizations of the Doppler blood flow signal. It can be found that this correction method has greatly improved the spectral width estimation accuracy. Fig. 4 (A, C) and Fig.4 (B, D) show the normalized root-mean-square errors of the RMS spectral width waveforms extracted from the signals with the constant RMS width of 100 Hz and with the varying RMS width respectively. It can be found that both uncorrected and corrected normalized root-mean-square errors of the RMS spectral width waveforms extracted from the signals with the constant RMS width are less than those with the varying RMS width. The normalized root-mean-square errors of the RMS spectral width waveforms using the 5 ms analysis window duration shown in Fig. 4 (A, B) is much larger than the corresponding ones using the 10 ms analysis window duration shown in Fig. 4 (C, D).

A simple consideration of the Doppler signal obtained for the arterial flow indicates that the assumption of approximate stationarity of the Doppler signal statistics during time segment durations used for spectral analysis is unreasonable. The major theoretical advantage of the WT lies in its flexible length of analysis window, which is wide to analyze low frequencies and narrow to analyze high frequencies. This property is particularly suitable for signals having a relatively wide bandwidth and changing rapidly with time. However there is overestimated spectral broadening in the spectrum estimated using the WT according to (12). This overestimated RMS spectral width of the Doppler blood flow signals can be separately identified as the effects of window duration broadening and nonstationarity broadening. The effects of window broadening and nonstationarity broadening are different at various frequencies or scales because of the flexible analysis window duration. In high frequencies, since the window duration used is short, the spectral broadening for the WT is mainly window induced. While in low frequencies, the duration of the window is many times larger than that in high frequencies, the window induced broadening is reduced while the nonstationarity broadening is increased. Using the mean RMS width defined in (11), the increases in the RMS spectral width can be calculated and the spectral width estimation based on the WT can be corrected. In present study, to evaluate the performance of the correction methods, the simulated Doppler signal with known characteristics is used in the experiment since the true spectra width of a clinical Doppler signal is unknown. The results show that when using (12) to correct the estimation of the RMS spectral width in practice, the measurement accuracy of the slop of the mean frequency variation β is important.

V Conclusion

Despite the WT can generate the time-frequency representation and the mean frequency waveform estimation with better precise than the STFT does, the estimated spectral width of arterial Doppler signals using the WT may also include significant window and nonstationarity broadening errors and the nonstationarity broadening is dominated. When the frequency of the signal varies linearly with time, a closed form expression for the window and the nonstationary RMS spectral width could be obtained. A novel method based on the mean root-mean square-error (RMS) spectral width has been proposed to estimate the broadening errors in the WT spectrum. This correction method is proved useful to improve the spectral width estimation accuracy when using the WT to estimate the Doppler blood flow spectrum.

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