Frequency Band Energy Leakage in Wavelet Analysis and Fuzzy Processing Method

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Abstract

The frequency characteristics of the Daubechies *N* wavelet series are analyzed. The characteristics of the frequency band and the influences of the frequency band energy leakage (FBEL) of wavelet are expounded. The smoothness degree and the distinguishing frequency capability of the Daubechies wavelet series, and the frequency band energy (FBE) of the signal increase conformably with the increase of the parameter *N*. However, the degree of the FBEL decreases with the increase of the parameter *N*. An adjacent frequency bands energy increments comparison method (AFBEICM) is put forward, and this method accords with the energy change rules of the adjacent frequency bands. With the AFBEICM the interference of the FBEL is eliminated, and the feature extraction is optimized. In the basis of the AFBEICM, the frequency bands of the wavelet analysis are subdivided by a fuzzy processing method. It is the basis of the fuzzy decision of the system.
Keywords: Wavelet analysis, energy leakage, fuzzy decision, feature extraction

I. Introduction

In the sight of the filter groups of the multi-resolution analysis, the low pass filters and the high pass filters aren’t ideal, so the frequency characteristics of the frequency bands aren’t ideal. When one frequency component of the signal changes, the FBE of this frequency component’s frequency band changes. At the same time, the FBE of the left and right adjacent frequency bands of the above frequency band change also. It is shown that the FBEL phenomenon exists in the wavelet analysis and the wavelet packet analysis [1], [2]. The FBEL problem gives bad influences to the veracity of the signal feature extraction. If the feature extraction is used in the faults diagnosis systems, the above problem will result in a worrying judgment and affect the reliability of systems.

The distinguishing frequency capability of the wavelet analysis is the frequency band. In the wavelet decomposition there are several detail levels. The frequency band of the higher frequency detail levels is wider than that of the lower frequency detail levels. Although in the wavelet packet analysis the frequency bands can be subdivided, the wavelet packet analysis is more complex than the wavelet analysis. In the same condition the implementing time of the wavelet packet analysis is longer than that of the wavelet analysis. It is a main disadvantage of the wavelet packet analysis in the real time signal processing. Therefore, in the real time signal processing the wavelet analysis is often used.

Because the FBEL in the wavelet analysis gives some disadvantages to the signal processing, it needs to be eliminated. In eliminating the FBEL process, it is found that using the FBEL we can subdivide the frequency bands. Although subdividing the frequency bands with the FBEL isn't better than the wavelet packet analysis, it is very simple and its effect is good.

In this paper, the frequency characteristics of the frequency bands in the wavelet analysis and the characteristics of the FBEL will be discussed. The AFBEICM will be put forward. Furthermore, how to eliminate the influences of the FBEL by the AFBEICM and optimize the signal feature extraction will be expounded. In this paper, the Daubechies wavelet series will be analyzed, and db$N$ is acted as the shortened form of the Daubechies2~10, $N = 2,3,\cdots,10$.

Ⅱ. Some Properties of the Daubechies Wavelets

In the multi-resolution analysis, the dilation equations of the scaling function $\phi(t)$ and the wavelet function $\psi(t)$ are

$$\phi(t) = \sqrt{2}\sum_n h_n \phi(2t-n) \tag{1}$$

$$\psi(t) = \sqrt{2}\sum_n g_n \phi(2t-n) \tag{2}$$

Where $\{h_n\}$ and $\{g_n\}$ are the transfer functions of the dilation equations, $g_n = (-1)^n h(1-n)$. In the frequency domain, the dilation equations are

$$\hat{\phi}(2\omega) = H(\omega)\hat{\phi}(\omega) \tag{3}$$

$$\hat{\psi}(2\omega) = G(\omega)\hat{\phi}(\omega) \tag{4}$$

Where $H(\omega) = \frac{1}{\sqrt{2}}\sum_n h_n e^{-in\omega}$, $G(\omega) = \frac{1}{\sqrt{2}}\sum_n g_n e^{-in\omega}$. Their periods are 2π. $H(\omega)$ and $G(\omega)$ accord with

$$\left|H(\omega)\right|^2 + \left|H(\omega+\pi)\right|^2 = 1 \tag{5}$$

$$\left|G(\omega)\right|^2 + \left|G(\omega+\pi)\right|^2 = 1 \tag{6}$$

Because $H(0) = 1, G(0) = 0$, it is known that $H(\omega)$ is the low pass filter and $G(\omega)$ is the high pass filter.

When the Daubechies wavelets were constructed,

$$H(\omega) = \left[\frac{1}{2}\left(1+e^{-i\omega}\right)\right]^N Q(e^{-i\omega}) \tag{7}$$

$$G(\omega) = e^{-i\omega}\overline{H}(\omega+\pi) \tag{8}$$

Where $Q(e^{-i\omega})$ is a real coefficient polynomial [3], [4], [5], [6]. The characteristics of the low pass filter $H(\omega)$ and the high pass filter $G(\omega)$ are decided by the parameter N.

The low pass filters of the db2 and db3 are respectively

$$\left|H(\omega)\right|^2 = \frac{1}{2} + \frac{9}{16}\cos\omega - \frac{1}{16}\cos 3\omega \quad \textbf{(9)}$$

$$\left|H(\omega)\right|^2 = \frac{1}{2} + \frac{75}{128}\cos\omega - \frac{25}{256}\cos 3\omega + \frac{3}{256}\cos 5\omega \quad \textbf{(10)}$$

It is shown that $H(\omega)$ and $G(\omega)$ of the db2 and the db3 aren't ideal filters. As the same reason, $H(\omega)$ and $G(\omega)$ of the db N aren't ideal filters, too [7]. Because the frequency characteristic of the frequency band has a close relation with that of $H(\omega)$ and $G(\omega)$, the frequency characteristics of the frequency bands aren't ideal. Therefore, the FBEL problem exists in every frequency band, and the degree of the FBEL correlates with the db N and the specific frequency band.

Ⅲ. Frequency Characteristics of Frequency Bands

For example, there is a sinusoidal signal

$$u(t) = U\sin(2\pi ft + \phi) \quad \textbf{(11)}$$

The sampling frequency is 40960Hz, and the total number of data points is 16384. The signal can be fully decomposed into 11 levels by the db3. From the higher frequency to the lower frequency, the frequency bands can be decomposed into the details d1~d11 and the approximation a11. The frequency ranges of the d9, d8 and d7 frequency bands are respectively 40~80Hz, 80~160Hz and 160~320Hz.

A. Frequency Characteristics of the Adjacent Frequency Bands

Let us consider $U = 0.5\,\text{V}$, $\phi = 0^\circ$, and sweep the signal frequency from 82Hz to 158Hz, where the step is 2Hz. Figure 1 shows the frequency characteristic curves of the d9, d8 and d7 frequency bands.

In this paper, the adjacent frequency band at the lower frequency side of one frequency band is called the left frequency band, and that at the higher frequency side of one frequency band is called the right frequency band. Therefore, the d9 frequency band and the d7 frequency band are respectively the left and right adjacent frequency bands of the d8 frequency band. In Figure 1, the vertical axes, which adopt the relative values, denote E_{d9}/E_{sum}, E_{d8}/E_{sum} and E_{d7}/E_{sum}, where E_{d9}, E_{d8} and E_{d7} respectively denote the FBE of the d9, d8 and d7 frequency bands, $E_{sum} = E_{d9} + E_{d8} + E_{d7}$. The frequency characteristics of other frequency bands are similar to that of the d9, d8 and d7 frequency bands.

B. The FBEL and Its Influences

In Figure 1, the frequency component belongs to the d8 frequency band when the signal frequency sweeps in the range of 82~158Hz. In this condition, because the FBEs of the d9 and d7 frequency bands aren't synchronously zero. It is shown that the FBEL phenomenon exists. As the same reason, this phenomenon also exists when the frequency component belongs to other frequency bands. The main influence of the FBEL phenomenon is that it can result in a wrong judgment when the features of signal are extracted. Hence, it is important that we should pay attention to the FBEL phenomenon.

For example, in Figure 1, when one frequency component is $f = 120$ Hz, there are

$$E_{d9}/E_{sum} = 2.2\% \quad \textbf{(12)}$$

$$E_{d8} / E_{sum} = 80.6\% \quad \textbf{(13)}$$

$$E_{d7} / E_{sum} = 17.2\% \quad \textbf{(14)}$$

Although the FBE of the d8 frequency band is bigger than the FBE of d7 and d9 frequency bands, the FBE of the d7 frequency band is obvious. If the FBE of d8 and d9 frequency bands aren't considered, it is possibly considered that the changing frequency component is in the d7 frequency band. It is obviously a wrong judgment. The right judgment is that the changing frequency component is in the d8 frequency band. The reason of the FBE change of the d7 frequency band is the FBEL of the d8 frequency band. Therefore, considering the FBEL problem is very important.

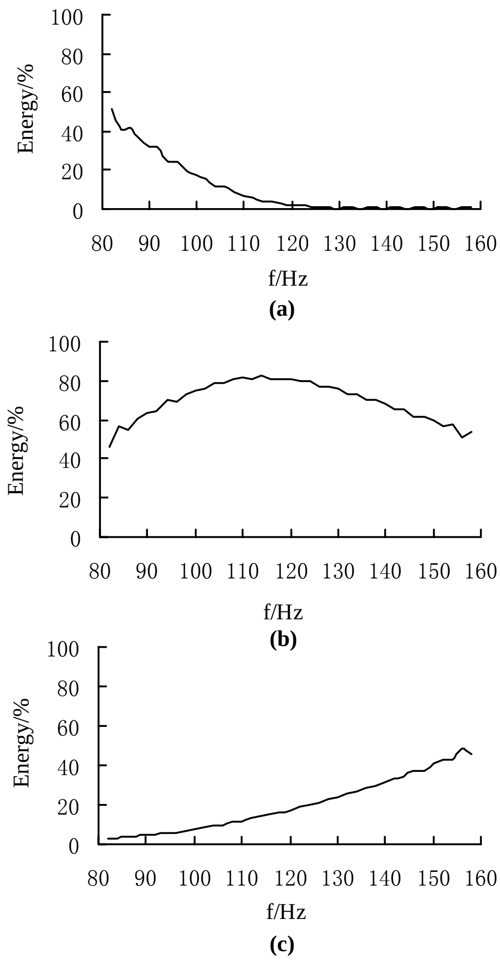

Fig. 1. The frequency characteristic curves of the d9, d8 and d7 frequency bands. (**a)** The frequency characteristic curve of the d9 frequency band. (**b)** The frequency characteristic curve of the d8 frequency band. (**c)** The frequency characteristic curve of the d7 frequency band.

C. Characteristics of the FBEL

In figure 1, Characteristics of the FBEL are as follows:

(1) When the frequency component is at the center of the d8 frequency band, the FBE of the d8 frequency band is close to the maximum, and the FBE of its left and right adjacent frequency bands are small. It is shown that the FBEL is small.

(2) When the frequency component leans to the right side of the d8 frequency band, the FBE of the d8 and d9 frequency bands decrease, but the FBE of d7 frequency band increases. It is shown that the FBEL concentrates to the d7 frequency band.

(3) Whereas, when the frequency component leans to the left side of the d8 frequency band, the FBE of the d8 and d7 frequency bands decrease, but the FBE of d9 frequency band increases. It is shown that the FBEL concentrates to the d9 frequency band.

Therefore, the FBEL of the d7 and d9 frequency bands reflect the departure degree of the frequency component from the center of the d8 frequency band. Other frequency bands have the same rules. Using these rules of the FBEL we can subdivide the frequency band, and judge the change range of the frequency component. It is the basic idea of the FBEL fuzzy processing method.

Ⅳ. The AFBEICM

When the state of the system changes, usually some frequency components change. If we only judge the values of the FBE, eliminating the influence of the FBEL is difficult. In this condition, we can use the absolute values of the adjacent FBE increments to analyze the FBEL. This method is called the AFBEICM.

Suppose there are k running states in the system. When the running state changes from the k_1th state to the k_2th state, a frequency component amplitude changes markedly, that is, there is a feature frequency component. In this condition, if the FBE of the dm frequency band changes from $E_{dm}(k_1)$ to $E_{dm}(k_2)$, its increment absolute value is denoted as

$$\Delta E_{dm} = \left| E_{dm}(k_2) - E_{dm}(k_1) \right| \quad \textbf{(15)}$$

Where ΔE_{dm+1}, ΔE_{dm} and ΔE_{dm-1} respectively denote the FBE increment absolute values of d$m+1$, dm and d$m-1$ frequency bands （if $m = 11$， d$m+1$ is replaced by a11）. The judgment rules are as follows, where ε denotes the threshold of the FBE increment absolute values, $m = 2 \sim 11$.

(1) If ΔE_{dm+1}, ΔE_{dm} and ΔE_{dm-1} are all less than ε， consider that the amplitude change of the feature frequency component can be ignored.

(2) If at least one of ΔE_{dm+1}, ΔE_{dm} and ΔE_{dm-1} are bigger than ε， we should do as follows:

When $\Delta E_{dm-1} > \Delta E_{dm}$ and $\Delta E_{dm-1} > \Delta E_{dm+1}$， the feature frequency component belongs to the d$m-1$ frequency band;

When $\Delta E_{dm} > \Delta E_{dm+1}$ and $\Delta E_{dm} > \Delta E_{dm-1}$， the feature frequency component belongs to the dm frequency band;

When $\Delta E_{dm+1} > \Delta E_{dm}$ and $\Delta E_{dm+1} > \Delta E_{dm-1}$， the feature frequency component belongs to the d$m+1$ frequency band.

The AFBEICM can eliminate the influence of the FBEL, and optimize the feature extraction of the signal. Though the above analysis is gotten in the db3 wavelet, the results can also be used in other wavelets.

Ⅴ. The FBEL Fuzzy Processing Method

Using the AFBEICM we can judge the frequency band to which the feature frequency component belongs. Suppose the feature frequency component belongs to the dm frequency band, using ΔE_{dm+1}, ΔE_{dm} and ΔE_{dm-1}, we can subdivide the dm frequency band to several frequency subbands (FSB). In this paper, the dm frequency band can be subdivided to five FSBs. From the lower frequency to the higher frequency, the five FSBs are respectively called FSB1, FSB2, FSB3, FSB4, FSB5.

Because the FBE increments have a close relation with the work conditions, it is difficult to accurately judge the FSB to which the feature frequency component belongs. We can only decide the degree of it. Therefore, it is more reasonable to judge the FSB to which the feature frequency component belongs using the fuzzy processing method. The fuzzy processing method of the FBEL can act as the basis of the fuzzy decision of the system.

A. The Fuzzy Variables

From above discussion, ΔE_{dm} reflects the departure degree of the feature frequency component from the dm frequency band center. ΔE_{dm+1} and ΔE_{dm-1} reflect the departure degree and direction of the feature frequency component from the dm frequency band center. Therefore, in this paper, d1~d10 frequency bands are processed by fuzzy technique. In the d1 frequency band, e_1 and e_2 act as inputs, where

$$e_1 = \Delta E_{d1} / \Delta E_{sum} \qquad \textbf{(16)}$$

$$e_2 = -\Delta E_{d2} / \Delta E_{sum} \qquad \textbf{(17)}$$

$$\Delta E_{sum} = \Delta E_{d2} + \Delta E_{d1} \qquad \textbf{(18)}$$

In d2～d10 frequency bands, e_1 and e_2 act as inputs, where

$$e_1 = \Delta E_{dm} / \Delta E_{sum} \qquad \textbf{(19)}$$

$$e_2 = (\Delta E_{dm-1} - \Delta E_{dm+1}) / \Delta E_{sum} \qquad \textbf{(20)}$$

$$\Delta E_{sum} = \Delta E_{dm+1} + \Delta E_{dm} + \Delta E_{dm-1} \qquad \textbf{(21)}$$

The output variable u denotes the FSB.

In the domain processing, dispersing e_1, e_2 and u we get the input variables e_1^*, e_2^* and the output variable u^*. The domains of e_1^* and e_2^* are

{-5,-4,-3,-2,-1,0,1,2,3,4,5}

The domains of u^* is {-2,-1,0,1,2}. The language values e_1^* and e_2^* are evenly divided into seven grades: negative large (NL), negative middle (NM), negative small (NS), zero (Z), positive small (PS), positive middle (PM), and positive large (PL). The language value u^* is evenly divided into five grades: NL, NS, Z, PS, and PL.

B. The Fuzzy Control Principle and the Fuzzy Decision

A table of the membership function for variables can be obtained by experiences. The fuzzy control principle is shown in Table 1.

Table 1

Fuzzy control principle table

u^*		e_1^*						
		NL	NM	NS	Z	PS	PM	PL
e_2^*	NL	NL	NL	×	×	×	×	×
	NM	×	NL	NL	NL	×	×	×
	NS	×	×	×	NS	NS	NS	×
	Z	×	×	×	×	×	NS	Z
	PS	×	×	×	PS	Z	Z	×
	PM	×	PL	PS	PS	×	×	×
	PL	PL	PL	×	×	×	×	×

×—It denotes that its condition is impossible.

The fuzzy decision of output can be achieved through a weight average decision method. The output decision is

$$y^* = \frac{\sum_{i=1}^{n} \mu_i y_i}{\sum_{i=1}^{n} \mu_i} \tag{22}$$

Where y_i is the support value of the fuzzy subclass U_i, and μ_i is the grade of membership of $y_i \in U_i$. The appropriate weight is selected according to design and experiences. The fuzzy control look-up table is obtained through corrections. From the fuzzy control look-up table a fuzzy variable can be obtained.

Ⅵ. Tests and Results

In this paper, the motor running noise of AC-AC variable frequency speed regulation system is acted as the research object. Digging the deep information from the running noise provides service to the fault diagnosis of the speed regulation system. In this system, the sampling frequency is 44100Hz; the total number of data points is 16384; the signal is decomposed into 11 levels by the db3, where the frequency range of the d9~d7 frequency bands is 43~344Hz.

It is shown that the FBE of the d9~d7 frequency bands of the running noise increases conformably as the load of motor increases by tests. For example, the load of motor changes from 30% rated load to 60% rated load, i.e., the work state of motor changes from state 1 to state 2. In this condition, the running noise is analyzed, and the result is shown in Table 2, where used the maximum of the FBE increments as the benchmark the FBE increments are standardized.

Table 2

FBE increments of frequency bands

Frequency bands	d11	d10	d9	d8	d7	d6	d5
FBE increments (%)	1.1	-0.9	30.6	100	7.2	6.0	-5.9

In Table 2, we can know, from the AFBEICM the feature frequency component belongs to the d8 frequency band, and the FBE increments of the d9 and d7 frequency bands are mainly caused by the FBEL of the d8 frequency band. Because the FBE increments of the d9 and d7 frequency bands are easily interfered by other frequency bands, the FBE increment of the d8 frequency band acted as the feature of the running noise change is more credibility. It can be seen that considering the FBEL influences can optimize the feature extraction and improve the reliability of systems.

Using the AFBEICM we knew that the feature frequency component belongs to the d8 frequency band. In the basis of this result, using the above fuzzy processing method we can know that the feature frequency component belongs to the FSB1 of the d8 frequency band. It is shown that the fuzzy processing method can accurately localize the feature frequency component.

Ⅶ. Conclusions

(1) The FBEL exists in the Daubechies wavelet series and other type wavelets. It affects the reliability of systems.

(2) From Daubechies2 to Daubechies10, the smoothness degree and the distinguishing frequency capability of the Daubechies wavelet series increase, and the degree of the FBEL decreases. The interference of the FBEL can be eliminated and the feature extraction can be optimized with the AFBEICM.

(3) Fully using the characteristics and the FBEL of the frequency bands the FBEL fuzzy processing method can subdivide the frequency bands of the wavelet analysis. It makes the basis of the fuzzy decision of the system.

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