Spatial Relations between Physical Objects with Property of Container in Virtual Environment

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Abstract

A physical object, a basic component of virtual environment, could be categorized as a solid object or a hollow object according to existence of inner space in the topological viewpoint. Also, a solid object has container property if it provides an inner space in cognitive viewpoint. These objects could be spatially related to each and such relations could be used as factors for development of the events and inferencing of the agents. To explicitly represent qualitative spatial relations, we develop mechanism based on topology, which provides a mathematical framework. In this paper, we define the physical object, solid object and hollow object using the point-set topology, and enhance the 4-intersection model into a new intersection model. Also, we define container property and specialize the spatial relations.
Keywords: physical object, solid object, hollow object, container, spatial relation

I. Introduction

The Physical object, a basic component of the virtual environment, occupies some spatial range in a space and provides spaces that other objects can reside in.[1,2] It could be categorized as a solid object or a hollow object according to existence of inner space in topological viewpoint. A solid object only provides an inner space, and a hollow object provides both an inner space and an outer space.[3] A solid object has *container* property if it provides an inner space in the cognitive viewpoint.[4] Also, these objects could be spatially related to each and such relations could be used as factors for development of the events and inferencing of the agents.[5] We develop mechanism for explicit representation of spatial relations.[6] The mechanism is based on topology, which provides a mathematical framework for qualitative representation of spatial relations.[7]
Researches for topological spatial relations have been conducted in various areas. In GIS, the 4-intersection model was developed to specify topological relations between 2-D regions such as *disjoint, touch, equal overlap, coveredby* and *contain*.[8] However, the model is not suitable for representing spatial relations between 3-D physical objects with the constraint that they cannot occupy the same space range. In Virtual Reality, researches have been performed such a way that the set of topological relations from the GIS is stripped down to one with fewer relations, i.e., *disjoint, touch, contain, equal*.[9] Therefore, we need to establish a new topological spatial relation model applicable to physical objects in 3-D space. However, we cannot define container as a special case of

the solid object only with the concept of topology. Therefore, we need a method that transforms the solid object and judges if it is a *container* based on topology.

In this paper, we propose a spatial relation model applicable to physical objects with the *container* property. We define the physical object, solid object and hollow object using the point-set topology, and enhance the 4-intersection model into a new intersection model. Among the spatial relations identified from the intersection model, we evaluate the *touch* relations involving a solid object to see if the object has *container* property. Those spatial relations with *container* object are specialized accordingly.

II. Point-set topology

We use the notion of point-set topology to establish a mathematical framework for spatial relations. This section presents basic concepts of topology found in relevant papers and text books without proof.[8,10,11]

Definition 1 A topology on a set X is a collection $\Im$ of subsets of X having the following properties:

(1) The empty set $\varnothing$ and set X belong to $\Im$;
(2) The union of the elements of any subcollection of $\Im$ is in $\Im$;
(3) The intersection of the elements of any finite subcollection of $\Im$ is in $\Im$.

Definition 2 Given $A \subset X$, the interior of A, denoted by A°, is defined as the union of all open sets that are contained in A.

Definition 3 The closure of A, denoted $\overline{A}$, is defined as the intersection of all closed sets that contains A.

Definition 4 The boundary of A, denoted by ∂A, corresponds to the intersection between the closure of X and the complement of X, i.e., $\partial A = \overline{A} \cap \overline{X - A}$.

Definition 5 The exterior of a A, denoted by A^V, corresponds to $A^V = X - \overline{A}$.

Definition 6 Given $Y \subset X$, A separation of Y is a pair A, B of subsets of X satisfying the following three conditions:

(1) $A \neq \varnothing$ and $B \neq \varnothing$
(2) $A \cup B = Y$
(3) $\overline{A} \cap B = \varnothing$ and $A \cap \overline{B} = \varnothing$

If there exists a separation of Y then Y is said to be disconnected, otherwise Y is said to be connected.

Proposition 1 $A^\circ \cap \partial A = \varnothing$.

Proposition 2 $A^\circ \cup \partial A = \overline{A}$.

Proposition 3 If A, B form a separation of Y and if Z is a connected subset of Y then either $Z \subset A$ or $Z \subset B$.

Proposition 4 Assume $Y \subset X$. If $Y^\circ \neq \varnothing$ and $\overline{Y} \neq X$, then Y° and $X - \overline{Y}$ form a separation of $X - \partial Y$, and thus ∂A separates X .

III. Definition of space and physical object in topological space

We aim to elaborate spatial relations between physical objects with container property in 3-D virtual environment. We first define physical objects and investigate topological spatial relations between them. To apply the concept of topology, we assume that the topological space X is connected topological space.[8,10] Also, we define physical object, solid object and hollow object by using point-set topology.[8]

Assumption 1. The entire space X is a connected point-set topological space.

A. *Definition of physical object*

A physical object resides to occupy space in virtual environment.[1,2] This object has the constraint that two objects cannot occupy same space range.[2] The definition of Physical object is as follows.

Definition 7 $A \subset X$ **,** A is called an physical object satisfying the following conditions:

(1) A non-empty proper subset A of X satisfying A° is connected and $A = \overline{A^\circ}$.

(2) Let A , B be a set satisfying condition (1). If $A^\circ \cap B^\circ = \varnothing$ always, A , B are physical objects.

Proposition 5 If A is a physical object in X , then $\partial A \neq \varnothing$.

Proof: $A^\circ \neq X$, $A^\circ \neq \varnothing$, $A^\circ = \overline{A}$ by Definition 7.1. From Proposition 4 it follows that A° and $X - A$ form a separation of $X - \partial A$. If $\partial A = \varnothing$ then the two sets would form a separation of X , which is impossible since X is connected; therefore, $\partial A \neq \varnothing$.

Proposition 6 If A, B are physical objects in X , then $A^\circ \cap \partial B = \varnothing$, $\partial A \cap B^\circ = \varnothing$.

Proof: Assume that $A^\circ \cap \partial B \neq \varnothing$. By Proposition 2 $A^\circ \cup \partial A = \overline{A}$ and $A^\circ \cup \partial (A^\circ) = \overline{A^\circ}$. $A^\circ \cup \partial A = A^\circ \cup \partial (A^\circ)$ since $\overline{A} = A = \overline{A^\circ}$. Furthermore, by Proposition 1 $A^\circ \cap \partial (A^\circ) = \varnothing$ and $A^\circ \cap \partial A = \varnothing$. In consequence $\partial (A^\circ) = \partial A$. If $x \in \partial A \cap B^\circ$ then $x \in \partial (A^\circ)$ and $A^\circ \cap B^\circ \neq \varnothing$ since B° is open and contains x, which is impossible since A , B are physical objects; therefore, $A^\circ \cap \partial B = \varnothing$. Clearly, $\partial A \cap B^\circ = \varnothing$ and the result follows.

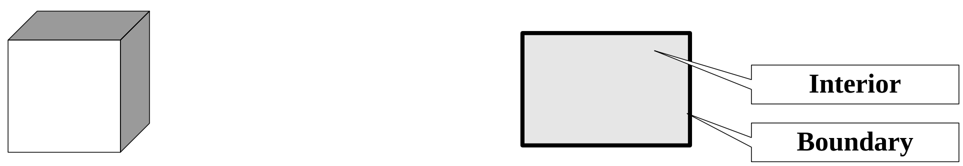
Proposition 7 If A, B are physical objects in X , then $A^\circ \subset X - \overline{B}$ and $B^\circ \subset X - \overline{A}$.

Proof: Assume that A , B are physical objects in X . By proposition 2 $X = (X - \overline{B}) \cup \overline{B}$ $= (X - \overline{B}) \cup B^\circ \cup \partial B$. By assuming that A is physical object in X , $A^\circ \subset (X - \overline{B}) \cup B^\circ \cup \partial B$ since $A^\circ \subset \overline{A} \subset X$. By definition 7 and proposition 6 $A^\circ \subset (X - \overline{B})$ since $A^\circ \cap B^\circ = \varnothing$ and $A^\circ \cap \partial B = \varnothing$. Clearly, $B^\circ \subset X - \overline{A}$ and the result follows.

B. *Classification of physical object*

A physical object provides spaces internally in its inside or externally on its surface. Inner space and outer space is regarded as different spaces in topological viewpoint. Assuming that object A provides both inner space and outer space, object B resides in inner space and object C resides in outer space, the relation between A and B is different from the relation between A and C in topological viewpoint. From this point of view physical objects are categorized into solid objects and hollow objects according to existence of inner space.[3] A solid object is a dense object that offers outer space and doesn't offer inner space- e.g., pencil, table and chair. The solid object is defined as follows.

Definition 8 $A \subset X$, A is called a solid object if A is a physical object and exterior of A (i.e., $X - \overline{A}$) is connected. We use the concept of the interior and the boundary from point-set topology without redefinition. (Fig. 1)



(a) Object in virtual environment **(b) Object in topological space**

Fig. 1. Solid Object

A hollow object is an empty object that offers both outer space and inner space -e.g., box, room. The hollow object is defined as follows.

Definition 9. $A \subset X$, A is called a hollow object if A is a physical object and the exterior of A (i.e., $X - \overline{A}$) is disconnected. If $X - \overline{A}$ is disconnected, it follows that there exists separation of $X - \overline{A}$, i.e., there exists a pair of subsets of $X - \overline{A}$ satisfying Definition 6 This pair of subsets is defined as the inner space and the outer space respectively. Also, it follows that the boundary is disconnected. If the boundary is disconnected, it follow that there exists separation of the boundary, i.e., there exists a pair of subsets of the boundary satisfying Definition 6 This pair of subsets is defined as the inner space and the outer space respectively.

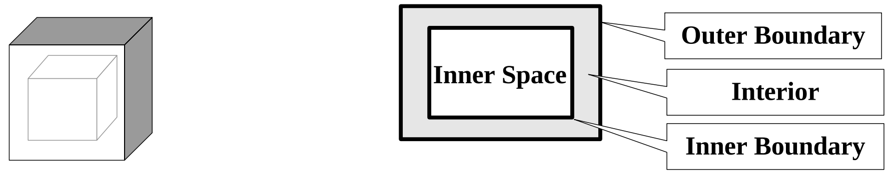
(1) Inner Space(^{is})
The exterior of A (i.e., $X - \overline{A}$) satisfying that All connected lines between Point-set within the exterior of A and point-set within ∂X pass through A°

(2) Outer Space(^{os})
$X - \overline{A} - \overline{X^{is}}$

(3) Inner Boundary(∂^{i})
$\overline{A} \cap \overline{A^{is}}$

(4) Outer Boundary(∂^{o})
$(X - \overline{A^{is}}) \cap \overline{X - A} \cap \overline{A}$

Fig. 2 represents hollow object.



(a) object in virtual envrionment **(b) object in topological space**

Fig. 2. Hollow Object

Proposition 8. If A, B are physical objects in X and A is a hollow object, then either $B^{\circ} \subset A^{is}$ or $B^{\circ} \subset A^{os}$. Also, if B is a hollow object then either $A^{\circ} \subset B^{is}$ or $A^{\circ} \subset B^{os}$.

Proof: Assume that A, B are physical objects in X and A is a hollow object. By Proposition 7 $B^{\circ} \subset X - \overline{A}$. Also, by Definition 9 and Proposition 3 either $B^{\circ} \subset A^{is}$ or $B^{\circ} \subset A^{os}$. Clearly, either $A^{\circ} \subset B^{is}$ or $A^{\circ} \subset B^{os}$ and the result follows.

IV. Definition of space and physical object in topological space

We propose a new intersection model to elaborate topological relations among physical objects. We consider the intersection model between two solid objects, between solid object and hollow object and between two hollow objects, since each type has a different set of constraints applicable to the spatial relations. The intersection model between respective objects is represented as follows (Table 1, Fig. 3, Table 2, Fig. 4, Table 3, Fig. 5)

Table 1. Relations between two Solid Objects

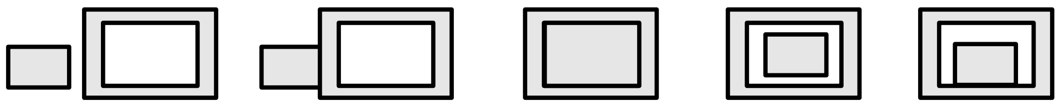
	$\partial \cap \partial$	
$R_{ss}1$	Ø	A and B are disjoint
$R_{ss}2$	¬Ø	A and B touch

(a) $R_{ss}1$ **(b) $R_{ss}2$**

Fig. 3. Relation between two Solid Objects

Table 2. Relations between Solid Object and Hollow Object

	$\partial^{o} \cap \partial$	$\partial^{i} \cap \partial$	$^{is}\cap \partial$	$^{is}\cap ^{\circ}$	
$R_{hs}1$	Ø	Ø	Ø	Ø	A(hollow) and B(solid) are disjoint
$R_{hs}2$	¬Ø	Ø	Ø	Ø	Outer boundary of A(hollow) and boundary of B(solid) touch.
$R_{hs}3$	Ø	¬Ø	Ø	¬Ø	B(solid) fits inner space of A(hollow).
$R_{hs}4$	Ø	Ø	¬Ø	¬Ø	B(solid) is contained inner space of A(hollow).
$R_{hs}5$	Ø	¬Ø	¬Ø	¬Ø	Inner boundary of A(hollow) and boundary of B(solid) touch.

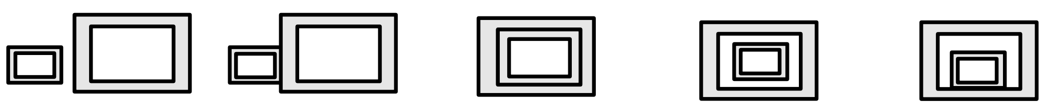


(a)$R_{hs}1$ **(b)$R_{hs}2$** **(c)$R_{hs}3$** **(d)$R_{hs}4$** **(e)$R_{hs}5$**

Fig. 4. Relations between Solid Object and Hollow Object

Table 3. Relations between two Hollow Objects

	$\partial^o\cap\partial^o$	$\partial^o\cap\partial^i$	$\partial^o\cap{}^{is}$	$^o\cap{}^{is}$	$\partial^i\cap\partial^o$	$\partial^i\cap{}^{is}$	$^{is}\cap\partial^o$	$^{is}\cap{}^o$	$^{is}\cap\partial^i$	$^{is}\cap{}^{is}$
$R_{hh}1$	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø
$R_{hh}2$	¬Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø
$R_{hh}3$	Ø	¬Ø	Ø	¬Ø	Ø	¬Ø	Ø	Ø	Ø	¬Ø
$R_{hh}4$	Ø	Ø	¬Ø	¬Ø	Ø	¬Ø	Ø	Ø	Ø	¬Ø
$R_{hh}5$	Ø	¬Ø	¬Ø	¬Ø	Ø	¬Ø	Ø	Ø	Ø	¬Ø
$R_{hh}6$	Ø	Ø	Ø	Ø	¬Ø	Ø	Ø	¬Ø	¬Ø	¬Ø
$R_{hh}7$	Ø	Ø	Ø	Ø	Ø	Ø	¬Ø	¬Ø	¬Ø	¬Ø
$R_{hh}8$	Ø	Ø	Ø	Ø	¬Ø	Ø	¬Ø	¬Ø	¬Ø	¬Ø
$R_{hh}1$	A(hollow) and B(hollow) are disjoint									
$R_{hh}2$	Outer boundary of A(hollow) and Outer boundary of B(hollow) touch									
$R_{hh}3$	A(hollow) fits inner space of B(hollow).									
$R_{hh}4$	A(hollow) is contained inner space of B(hollow)									
$R_{hh}5$	Inner boundary of B(hollow) and Outer boundary of A(hollow) touch.									
$R_{hh}6$	B(hollow) fits inner space of A(hollow).									
$R_{hh}7$	B(hollow) is contained inner space of A(hollow)									
$R_{hh}8$	Inner boundary of A(hollow) and Outer boundary of B(hollow) touch.									



(a)$R_{hh}1$ **(b)$R_{hh}2$** **(c)$R_{hh}3$ or $R_{hh}6$** **(d)$R_{hh}4$ or $R_{hh}7$** **(e)$R_{hh}5$ or $R_{hh}8$**

Fig. 5. Relations between Hollow Objects

V. Definition of container and specialization of spatial relations

A. *Definition of container property*

Development of events or planning of agents varies according to the *touch* patterns of the objects. Consider, for instance, a situation that a book and a notebook touch each other and a nail is studded into the book. Given an order that the book is separated from the notebook and the nail, the agent would plan based on spatial relations in the situation. If the agent is aware of the topological relations, the relation between the book and notebook and that between the book and nail are considered identical. As a result, it is obvious the agent cannot plan appropriate actions. He/She needs to elaborate the spatial relations according to *touch* patterns. To this elaboration, we introduce container. While a container is defined as one with capacity to allow objects inside in other papers, [12,13] we redefine container as follows.

Definition 10 As illustrated in Fig. 6, a solid object is called an object having container property if an object covered by panel has the property of a hollow object.

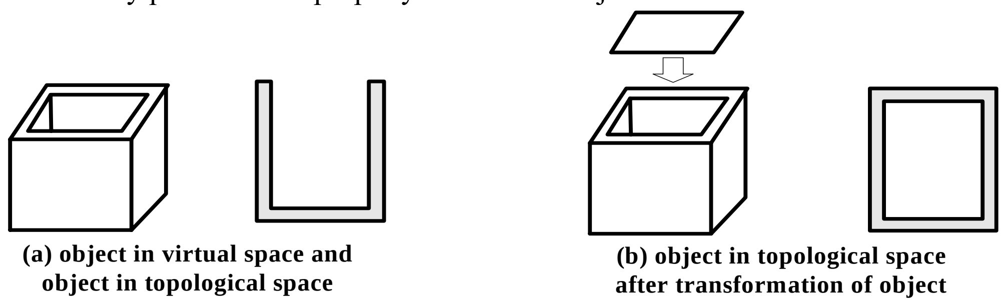


Fig. 6. Solid Object having property of Container

A container offers an inner space in cognitive viewpoint[4], but it doesn't provide a inner space in the topological viewpoint -e.g., cup, book shelf and bottle. The object has *container* property if the solid object is transformed into a container by another object. For example, a transformed part of a book has the *container* property in case a nail is studded into the book. The method for checking *container* property is as follows.

Conditions:

(1) The target object regards to the object subject to transformation
(2) Some panel can become a part of the target object.
(3) The panel is of the same size as that of the entrance of the container and has minimum thickness.

Operations:

(1) Transform the solid object by placing the panel on the object in parallel with the entrance of the container.
(2) Check if it is a hollow object.

B. *Spatial relations between objects having container property*

We assume that solid object A is the target object and denote *touch* relation ($R_{ss}2$, $R_{hs}2$) by R_{b_touch}. These relations are to be specialized by *container* property of the target object.
The method for specialization is as follows.

(1) Check if the touched part of solid object has container property.
(2) Check if the panel with the same size as that of the entrance of the container and the minimum thickness can cover the entrance of the container.
(3) If it can cover, apply the intersection model with covered. If not, reduce the length of the residing object along the direction of the entrance, cover the entrance with panel and apply the intersection model

Fig. 7 shows specialization of R_{b_touch} and Fig. 8 shows examples.

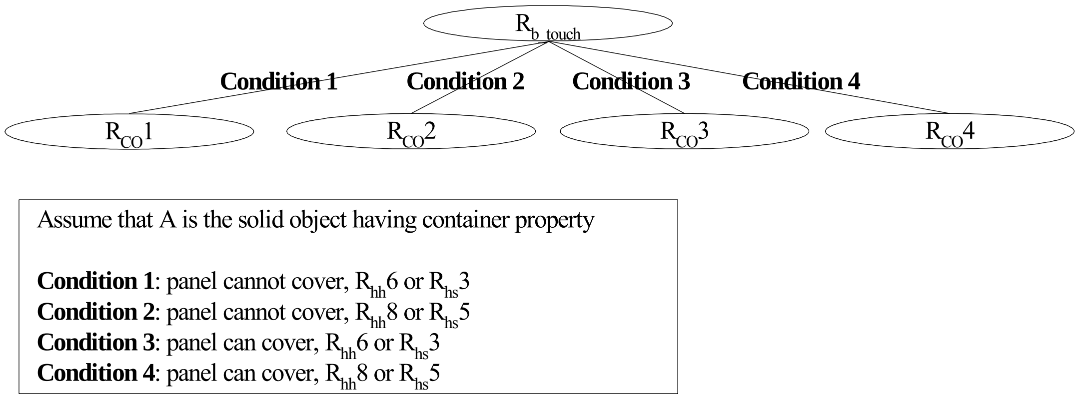


Fig. 7. Hierarchy of R_{b_touch}

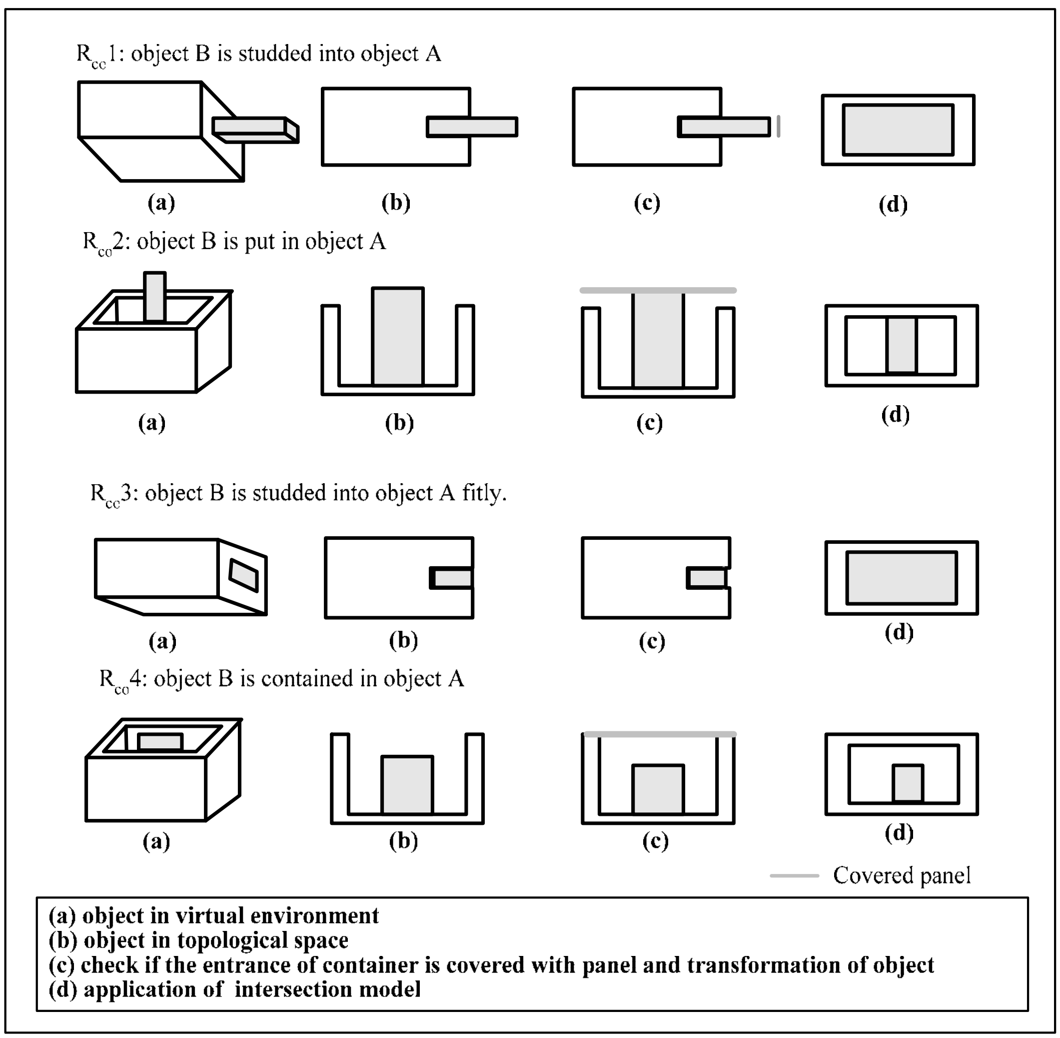


Fig. 8. Examples for spatial relations between a Solid Object having *Container* property and a Physical Object

VI. Conclusion

In this paper, we proposed a new intersection model, which is used as the foundation of constructing spatial relations among physical objects. We first defined the physical object, solid object and hollow object. We proved topological spatial relations to occur between physical objects using these concepts. This model provides a mathematical framework for qualitative representation of spatial relations between physical objects and assures completeness. To provide diverse spatial relations, occurring in reality, we have defined *container* property and accordingly specialized the spatial relations.

An ultimate form of spatial model should encompass diverse spatial relations meaningful to intelligent agents as well as topological relations. Along with providing diverse relations, we should also consider various forces such as the gravity and orientation of relations according to the viewpoints of agents.

Acknowledgements

This work was partially supported by the Basic Research Program of Korea Science and Engineering Foundation under the number of R05-2003-00011941-0, and the Brain Korea 21 (2005)

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