Numerical Analysis for Jump-Diffusion Differential Equations

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Abstract

In this paper, Euler approximation is introduced for a broad class of jump-diffusion equations, and the numerical approximation is studied for multidimensional jump-diffusion processes. We prove that numerical solutions based on the Euler scheme will converge to the analytic solution if both remain within a compact set. Both of analytic solution and the approximate solution are bounded in probability. The numerical solution is solved by this method, when the linear growth and global Lipschitz conditions are not satisfied by the system. An example is studied to illustrate the results.
Keyword: Jump-diffusion equations; Euler scheme; Numerical solution.

I. Introduction

We consider a n-dimensional jump-diffusion process $\{x(t)\}$ satisfying

$$dx(t) = \mu(t, x(t))dt + \sigma(t, x(t))dB_t + \int c(t, x(t), u)\tilde{v}(dt, du), \tag{1}$$

where $\mu(t,x)$ and $c(t,x,u)$ are R^n-valued and $\sigma(t,x)$ is $n\times m$-matrix valued for $x,u \in R^n$. $\{B_t\}$ is a standard m-dimensional Brownian motion, and

$$\tilde{v}(ds, dy) = v(ds, dy) - \Pi(dy)ds$$

is a compensated Poisson random measure on $[0.\infty)\times R^n$ which is independent of $\{B_t\}$. The solution $x(t)$ is interpreted as the stochastic integral

$$x(t) = x_0 + \int_0^t \mu(s, x(s))ds + \int_0^t \sigma(s, x(s))dB_s + \int_0^t \int c(s, x(s), u)\tilde{v}(ds, du). \tag{2}$$

In mathematical finance theory, one of the principal interest is focused on option pricing. Among the earliest investigations of continuous-time models of asset prices with jumps is the work of Merton[1]. In particular, jump-diffusion models described as Ito process disturbed by Poisson process or random measure are general enough to include most interesting cases that may arise. These models are discussed by Jeanblanc-Picque[2] and Sabrina[3]. In general, system (1) is analytically intractable. Most existing proofs of the convergence of the such numerical schemes rely on Global Lipschitz and Linear growth conditions and we here mention see Maghsoodi[4] and Kloeden[5]. Unfortunately there conditions are often not met by system of interest. Federico[6] and Protter[7] relaxed the global Lipschitz condition and Linear growth condition. However, both results provide no information on the order of approximation. In this paper, we show that under certain conditions, weaker than Global Lipschitz and Linear growth conditions, the Euler scheme applied to system (1), converges to the analytic solution $x(t)$, and in doing so bound the order of this approximation.

II. The Euler approximation

For system (1) the discrete time Euler approximation on $t \in \{0, \Delta t, \cdots, N\Delta t = T\}$ is given by the iterative scheme

$$x_{\Delta t}(t+\Delta t) = x_{\Delta t}(t) + \mu(t, x_{\Delta t}(t))\Delta t + \sigma(t, x_{\Delta t}))\Delta B_t + \int c(t, x_{\Delta t}(t), u)\tilde{v}(\Delta t, du) \quad (3)$$

with initial value $x_{\Delta t}(0) = x_0$. Here the time increment is Δt, and the $\Delta B_t = B(t+\Delta t) - B(t)$ represent N independent draws from an m-dimensional Normal distribution whose individual components have mean zero and variance Δt. Furthermore, we shall rewrite $x_{\Delta t}(t)$ as the integral

$$x_{\Delta t}(t) = x_0 + \int_0^t \mu(s, \hat{x}_{\Delta t}(s))dB_s + \int_0^t \int c(s, \hat{x}_{\Delta t}(s), u)\tilde{v}(ds, du) \quad (4)$$

where we have introduced the piecewise constant process

$$\hat{x}_{\Delta t} = \sum_{k=1}^{N} x_{\Delta t}((k-1)\Delta t) I_{[(k-1)\Delta t, k\Delta t]}(t) \quad (5)$$

and I_A is the indicator function for set A.

The solution to (1), $x(t)$ is a member of the open set $G \subseteq R^n$, for $t \in [0,T]$ and initial value $x_0 \in G$. Define $x_{\Delta t}(t)$ as the Euler approximation (3) and let $D \subseteq G$ be any compact set. Expression (4) extends the definition of the Euler scheme to all $t \in [0,T]$, and may also be expressed in the stochastic differential form

$$dx_{\Delta t}(t) = \mu(t, \hat{x}_{\Delta t}(t))dt + \sigma(t, \hat{x}_{\Delta t}(t))dB_t + \int c(t, \hat{x}_{\Delta t}(t), u)\tilde{v}(dt, du), \quad (6)$$

with initial condition $x_{\Delta t}(0) = x_0 \in G$.

Suppose $\int \Pi(du) < \infty$ the following conditions are satisfied

(i)(local Lipschitz condition)there exist a positive constant $K_1(D)$ such that $x, y \in D$

$$|\mu(t,x) - \mu(t,y)|^2 \vee \|\sigma(t,x) - \sigma(t,y)\|^2 \vee \int |c(t,x,u) - c(t,y,u)|^2 \Pi(du) \le K_1(D)|y-x|^2$$

(ii) there exists a C^2- positive function $V(\cdot): G \to R^+$ such that $\{x \in G : V(x) \le r\}$ is compact for any $r > 0$;

(iii) Let $\psi_1(t), \psi_2(t)$, be two continuous non-negative functions, and there exists a positive constant $K(D)$. Assume that for all $x \in G$, such that

$$LV(x) \le K(D) + \psi_1(t) + \psi_2(t)V(x)$$

Where

$$LV(x) \equiv V(x)\mu(t,x) + \tfrac{1}{2} trac[\sigma^T(t,x)V_{xx}(x)\sigma(t,x)] + \int \{V(x + c(t,x,u)) - V(x) - V_x(x)c(t,x,u)\}\Pi(du)$$

(iv) there exists a positive constant $K_3(D)$ such that for $x, y \in D$

$$|V(x) - V(y)| \vee |V_x(x) - V_y(y)| \vee |V_{xx}(x) - V_{yy}(y)| \le K_3(D)|x-y|$$

If condition (i) holds then there exists a positive constant $K_2(D)$ such that for $x, y \in D$

$$|\mu(t,x)|^2 \vee \|\sigma(t,x)\|^2 \vee \int |c(t,x,u)|^2 \Pi(du) \le K_2(D). \quad (7)$$

III. The main results

In this section, the objective is that under the condition described above, We will prove the following useful convergence result.

Theorem 1. If τ is the first exist time of either the solution $x(t)$ or the Euler approximate solution $x_{\Delta t}(t)$ from a bounded region D, and $\mu(t,x(t))$, $\sigma(t,x(t))$ and $c(t,x(t),u)$ satisfy conditions (i), then for $\Delta tT<1$

$$E\left[\sup_{0\le t\le\tau\le T}|x_{\Delta t}(t)-x(t)|^2\right]\le C_1(D)e^{C_2(D)T}\Delta t=C(D)\Delta t.$$

Thus, as long as $x_{\Delta t}(t)$ and $x(t)$ remain in D the Euler scheme $x_{\Delta t}(t)$ converges to the solution $x(t)$ of equation as $\Delta t\to 0$.

Proof. In order to prove this result, we consider only trajectories $x_{\Delta t}(t)$ and $x(t)$ which remain within a bounded region D. To achieve this, introduce the stopping time $\tau=\rho\wedge\theta$ where

$$\rho=\inf\{t\ge 0: x_{\Delta t}(t)\notin D\}\quad\text{and}\quad\theta=\inf\{t\ge 0: x(t)\notin D\}$$

are the first time that $x_{\Delta t}(t)$ and $x(t)$, respectively, leave D. We will define D more precisely later. Let $T_1\in[0,T]$ be an arbitrary time. We can derive that for any $t\in[0,\tau\wedge T_1]$

$$\begin{aligned}
&E[\sup_{0\le t\le\Gamma\wedge T_1}|x_{\Delta t}(t)-x(t)|^2]\\
&\le 3E\sup_{0\le t\le\Gamma\wedge T_1}\left|\int_0^t(\mu(s,\hat{x}_{\Delta t}(s))-\mu(s,x(s)))ds\right|^2\\
&+3E\sup_{0\le t\le\Gamma\wedge T_1}\left|\int_0^t(\sigma(s,\hat{x}_{\Delta t}(s))-\sigma(s,x(s)))dB_s\right|^2\\
&+3E\sup_{0\le t\le\Gamma\wedge T_1}\left|\int_0^t\int(c(s,\hat{x}_{\Delta t}(s),u)-c(s,x(s),u))\tilde{v}(ds,du)\right|^2
\end{aligned}\tag{8}$$

The Holder inequality shows that

$$3E\sup_{0\le t\le\tau\le T_1}\left|\int_0^t(\mu(s,\hat{x}_{\Delta t}(s))-\mu(s,x(s))ds\right|^2\le 3TE\int_0^{t\wedge T_1}|\mu(s,\hat{x}_{\Delta t}(s))-\mu(s,x(s))|^2\,ds\tag{9}$$

whence applying the Burkholder-Davis-Gundy inequality to the second term of (8) leads to

$$3E\sup_{0\le t\le\tau\wedge T_1}\left|\int_0^t(\sigma(s,\hat{x}_{\Delta t}(s))-\sigma(s,x(s))dB_s\right|^2\le 3C_1E\int_0^{\tau\wedge T_1}\|\sigma(s,\hat{x}_{\Delta t}(s))-\sigma(s,x(s))\|^2\,ds.\tag{10}$$

$$\begin{aligned}
&3E\sup_{0\le t\le\Gamma\wedge T_1}\left|\int_0^t\int(c(s,\hat{x}_{\Delta t}(s),u)-c(s,x(s),u))\tilde{v}(ds,du)\right|^2\\
&\le 3C_2E\int_0^{\tau\wedge T_1}\int|c(s,\hat{x}_{\Delta t}(s),u)-c(s,x(s),u)|^2\,\Pi(du)ds
\end{aligned}\tag{11}$$

where C_1 and C_2 are constants. If the coefficients of (1) are locally Lipschitz continuous (i.e. satisfy condition (i)), then since both $x(t)$ and $x_{\Delta t}(s)$ are bounded we may write

$$\begin{aligned}
&\mu(s,\hat{x}_{\Delta t}(s))-\mu(s,x(s))|^2\vee\|\sigma(s,\hat{x}_{\Delta t}(s))-\sigma(s,x(s))\|^2\\
&\vee\int|c(s,\hat{x}_{\Delta t}(s),u)-c(s,x(s),u)|^2du\le K_1(D)|\hat{x}_{\Delta t}(s)-x(s)|^2
\end{aligned}$$

for $s\in[0,\tau\wedge T_1]$. Substituting (9),(10) and (11),into (8) reveals that

$$\begin{aligned}
&E(\sup_{0\le s\le\tau\wedge T_1}|x_{\Delta t}(t)-x(t)|^2)\le 3K_1(D)(T+C_1+C_2)E\int_0^{\tau\wedge T_1}|\hat{x}_{\Delta t}(s)-x(s)|^2\,ds\\
&\le 6K_1(D)(T+C_1+C_2)E\int_0^{\tau\wedge T_1}|\hat{x}_{\Delta t}(s)-x_{\Delta t}(s)|^2\,ds\\
&+6K_1(D)(T+C_1+C_2)\int_0^{T_1}E\left[\sup_{0\le s'\le\tau\wedge s}|x_{\Delta t}(s')-x(s')|^2\right]ds.
\end{aligned}\tag{12}$$

Inspection of (5) reveals that $\hat{x}_{\Delta t}(s)=x_{\Delta t}([s/\Delta t]\Delta t)$, where $[s/\Delta t]$ is the integer part of $s/\Delta t$. We can now use (4) to show that

$$|\hat{x}_{\Delta t}(s)-x_{\Delta t}(s)|^2=|x_{\Delta t}([s/\Delta t]\Delta t)-x_{\Delta t}(s)|^2$$
$$\le 3K_2(D)\Delta t^2+3K_2(D)|B(s)-B([s/\Delta t]\Delta t)|^2 \qquad (13)$$
$$+|\int_{[s/\Delta t]\Delta t}^{s}\int c(v,x_{\Delta t}([s/\Delta t]\Delta t),u)\tilde{v}(dv,du)|^2$$

whence applying the Burkholder-Davis-Gundy inequality, condition (7) to the third term of (13) leads to

$$|\int_{[s/\Delta t]\Delta t}^{s}\int c(v,x_{\Delta t}([s/\Delta t]\Delta t),u)\tilde{v}(dv,du)|^2\le K_2(D)C_2\Delta t\,.$$

If $T\Delta t<1$ this inequality leads to

$$E\int_0^{\tau\wedge T_1}|\hat{x}_{\Delta t}(s)-x_{\Delta t}(s)|^2ds\le 3K_2(D)(mT+1+C_2)\Delta t\,. \qquad (14)$$

Using this result in (12) shows that

$$\mathrm{E}(\sup_{0\le t\le\tau\wedge T_1}|x_{\Delta t}(t)-x(t)|^2)\le C_1(D)\Delta t+C_2(D)\int_0^{T_1}E\left[\sup_{0\le r\le\tau\wedge s}|x_{\Delta t}(r)-x(r)|^2\right]ds$$

where $C_1(D)=18K_1(D)K_2(D)(T+C_1+C_2)(mT+1+C_2)$ and $C_2(D)=6K_1(D)(T+C_1+C_2)$.
On applying the Gronwall inequality we then have the following inequality

$$E\left[\sup_{0\le t\le\tau\le T}|x_{\Delta t}(t)-x(t)|^2\right]\le C_1(D)e^{C_2(D)T}\Delta t=C(D)\Delta t.$$

To proceed further we define the bounded domain

$$D=D(r)\equiv\{x\in G\quad \text{such that } V(x)\le r\}\,.$$

Theorem 2 If θ is the first exist time of the solution $x(t)$ to equation (1) from the domain $D(r)$,and a function $V(x)$ exists which satisfies conditions (ii) and (iii), then

$$P(\theta\ge T)\ge 1-\varepsilon\,.$$

Proof. We assume the existence of the non-negative function $V(x)$ satisfying condition (ii). Since $x(t)$ is governed by equation (1), applying $It\hat{o}'s$ formula to $V(x)$ yields

$$dV(x(t))\equiv LV(x(t))+V_x(x(t))\sigma(t,x(t))dB_t+\int[V(x+c(t,x,u))-V(x)]\tilde{v}(dt,du).$$

Integrating from 0 to $t\wedge\theta$ and taking expectations gives

$$E(V(x(t\wedge\theta)))\le V(x_0)+E\int_0^{t\wedge\theta}LV(x(s))ds.$$

Whence applying condition (iii) leads to

$$E(V(x(t\wedge\theta)))\le V(x_0)+E\int_0^{t\wedge\theta}(K(D)+\psi_1(s)+\psi_2(s)V(x(s))ds.$$

So by virtue of Gronwall lemma, we easily obtain that almost surely

$$E(V(x(t)))\le(V(x_0)+K(D)T+\int_0^T\psi_1(s)ds)\times\exp(\int_0^T\psi_2(s)ds)\,.$$

Let $M=(V(x_0)+K(D)T+\int_0^T\psi_1(s)ds)\times\exp(\int_0^T\psi_2(s)ds)$.
On noting that $V(x(\theta))=r$, since $x(\theta)$ is on the boundary of $D(r)$, the probability $P(\theta<T)$ can now be bounded as follows.

$$M\ge E[V(x(t\wedge\theta))]\ge E[V(x(\theta))I_{\{\theta<T\}}(\omega)]\ge rP(\theta<T) \qquad (15)$$

whence rearranging (15) leads to

$$P(\theta<T)\le M/r=\varepsilon. \qquad (16)$$

Here r can be made as large as required, for a given T and x_0, to accommodate any $\varepsilon\in(0,1)$. We note that the following useful result follows directly from Theorem 2.
Lemma 1. Let θ be the first exist time of the solution $x(t)$ to equation (1) from the domain $D(r)$,

and let the coefficients of (1) satisfy conditions (i). If a function $V(x)$ exists which satisfies conditions (ii) and (iii), then the limit of $\lim_{r\to\infty} D(r) \equiv G$ and, for $t \in [0,T]$ and $x_0 \in G$, $x(t)$ remains in G. Furthermore, $x(t)$ is the unique solution of equation (1) on $t \in [0,T]$ for all finite T.

Proof. Proof of this result can be found in paper of Mao[8].

We require a similar result to Theorem 2 for the Euler approximate solution $x_{\Delta t}(t)$.

Theorem 3. Let ρ be the first exit time of the Euler approximate solution (4) from the domain $D(r)$. Then if $\mu(x), \sigma(x)$ and $c(t,x,u)$ satisfy conditions (i) and there exists a function $V(x)$ which satisfies conditions (ii)-(iv), then (for sufficiently small Δt)

$$P(\rho \geq T) \geq 1 - \varepsilon(1 + \overline{H}(D)\Delta t^{1/2}).$$

Proof. Noting that $x_{\Delta t}(t)$ is the solution to (6). Applying the Ito formula to $V(x_{\Delta t}(t))$ and condition (iii) we obtain

$$\begin{aligned} dV(x_{\Delta t}(t)) &\leq K(D) + \psi_1(t) + \psi_2(t)V(x_{\Delta t}(t)) + \psi_2(t)[V(\hat{x}_{\Delta t}(t)) - V(x_{\Delta t}(t))]dt \\ &+ [V_x(x_{\Delta t}(t)) - V_x(\hat{x}_{\Delta t}(t))]\mu(t,\hat{x}_{\Delta t}(t))dt \\ &+ \tfrac{1}{2}\sigma^T(t,\hat{x}_{\Delta t}(t))(V_{xx}(x_{\Delta t}(t)) - V_{xx}(\hat{x}_{\Delta t}(t)))\sigma(t,\hat{x}_{\Delta t}(t))dt \\ &+ \int (V(x_{\Delta t}(t) + c(t,\hat{x}_{\Delta t}(t),u)) - V(\hat{x}_{\Delta t}(t) + c(t,\hat{x}_{\Delta t}(t),u))\Pi(du) \\ &- \int (V(x_{\Delta t}(t)) - V(\hat{x}_{\Delta t}(t)))\Pi(du) - \int (V_x(x_{\Delta t}(t)) - V_x(\hat{x}_{\Delta t}(t)))c(t,\hat{x}_{\Delta t}(t),u)\Pi(du) \\ &+ V_x(x_{\Delta t}(t))\sigma(t,\hat{x}_{\Delta t}(t))dB_t + \int [V(x_{\Delta t}(t) + c(t,\hat{x}_{\Delta t}(t),u)) - V(x_{\Delta t}(t))]\tilde{\nu}(dt,du). \end{aligned}$$

Integrating from 0 to $\rho \wedge t$ and taking expectations, and invoking (7) and (iv) leads to

$$\begin{aligned} &E[V(x_{\Delta t}(\rho \wedge t))] \\ &\leq V(x_0) + K(D)T + \int_0^T \psi_1(s)ds[K_2^{1/2}(D)(1 + (\int \Pi(du))^{1/2}) + 1/2K_2(D) + l + 2\int \Pi(du)] \\ &\times K_3(D)E\int_0^{\rho\wedge t} |\hat{x}_{\Delta t}(s) - x_{\Delta t}(s)|\, ds + E\int_0^t \psi_2(s)V(x_{\Delta t}(\rho \wedge s))ds \end{aligned}$$

where $l = \sup_{0\leq t\leq T} \psi_2(t)$

$$\int_0^{\rho\wedge t} E\,|\hat{x}_{\Delta t}(s) - x_{\Delta t}(s)|ds \leq [3K_2(D)(mT + 1 + C_2)T]^{\frac{1}{2}}\Delta t^{\frac{1}{2}}.$$

Follows from *Hölders* inequality and equation (14) for $t \in [0,T]$ and $\Delta t T < 1$.

$$\begin{aligned} &E[V(x_{\Delta t}(\rho \wedge t))] \\ &\leq V(x_0) + K(D)T + \int_0^T \psi_1(s)ds[K_2^{1/2}(D)(1 + (\int \Pi(du))^{1/2}) + 1/2K_2(D) + l + 2\int \Pi(du)] \\ &\times K_3(D)[3K_2(D)(mT + 1 + C_2)T]^{\frac{1}{2}}\Delta t^{\frac{1}{2}} + \int_0^t \psi_2(s)EV(x_{\Delta t}(\rho \wedge s))ds. \end{aligned}$$

Whence, on applying the Gronwall inequality

$$E[V(x_{\Delta t}(\rho \wedge t))] \leq (V(x_0) + K(D)T + \int_0^T \psi_1(s)ds)e^{\int_0^T \psi_2(s)ds} + H(D)\Delta t^{1/2}.$$

$$\begin{aligned} \textit{where } H(D) &= e^{\int_0^T \psi_2(s)ds}[K_2^{1/2}(D)(1 + (\int \Pi(du))^{1/2}) + \tfrac{1}{2}K_2(D) + l + 2\int \Pi(du)] \\ &\times K_3(D)[3K_2(D)(mT + 1 + C_2)T]^{\frac{1}{2}}. \end{aligned}$$

An argument analogous to that used to prove Theorem 2 can now be used to bound $P(\rho < T)$. Since $x_{\Delta t}(\rho)$ is on the boundary of $D(r)$ then $V(x_{\Delta t}(\rho)) = r$ which leads to

$$(V(x_0) + K(D)T + \int_0^T \psi_1(s)ds)e^{\int_0^T \psi_2(s)ds} + H(D)\Delta t^{1/2} \geq rP(\rho < T).$$

Defining $\overline{H} = H(D)e^{-\int_0^T \psi_2(s)} / (V(x_0) + K(D)T + \int_0^T \psi_1(s)ds$ reveals that

$$P(\rho < T) \le (1 + \overline{H}(D)\Delta t^{1/2})\varepsilon,$$

where ε is defined in equation (16). Hence our claim is proved.

The significance of Theorem 2 and 3 is that both $x(t)$ and $x_{\Delta t}(t)$ remain within the domain $D(r)$, and therefore by Theorem 1 the Euler scheme will converge to the $x(t)$, with probability

$$P(\tau < T) \le P(\rho < T) + P(s < T) \le (2 + \overline{H}(D)\Delta t^{1/2})\varepsilon. \tag{17}$$

Theorem 4. Let G be an open subset of R^n, and denote the unique solution of (1) for $t \in [0,T]$ given $x_0 \in G$ by $x(t) \in G$. Define $x_{\Delta t}(t)$ as the Euler approximation (3) and let $D \in G$ be any compact set. Suppose conditions (i)-(iv) are satisfied. Then for any $\varepsilon > 0, \delta > 0$ there exists $\Delta t^* > 0$ such that

$$P(\sup_{0 \le t \le T} | x_{\Delta t}(t) - x(t) |^2 \ge \delta) \le \varepsilon,$$

provided $\Delta t \le \Delta t^*$ and the initial value $x_0 \in G$.

Proof. Introducing the event sub-space $\overline{\Omega} = \{\omega : \sup_{0 \le t \le T} | x_{\Delta t}(t) - x(t) |^2 \ge \delta\}$, and using Theorem 1, we find that

$$\begin{aligned} C(D)\Delta t^2 &\ge E[\sup_{0 \le t \le T} | x_{\Delta t}(t) - x(t) |^2] \ge E[I_{\{\tau \ge T\}}(\omega) \sup_{0 \le t \le T} | x_{\Delta t}(t) - x(t) |^2] \\ &\ge \delta E[I_{\{\tau \ge T\}}(\omega) I_{\{\overline{\Omega}\}}(\omega)] \ge \delta[P(\overline{\Omega}) - P(\tau < T)]. \end{aligned}$$

Whence on using (17) we conclude that

$$P(\overline{\Omega}) = P(\sup_{0 \le t \le T} | x_{\Delta t}(t) - x(t) |^2 \ge \delta) \le 2\varepsilon + \varepsilon \overline{H}(D)\Delta t^{1/2} + \frac{C(D)}{\delta}\Delta t$$

which for appropriate choice of Δt, proves Theorem 4.

IV. Example

Consider the following stochastic equation

$$\begin{aligned} dx(t) &= diag(x_1(t), \cdots, x_n(t))[A(x(t) - \alpha)dt + D(x(t) - \alpha)dB(t)] \\ &\quad + diag(x_1(t), \cdots x_n(t)) \int \delta(u)\overline{v}(dt, du). \end{aligned} \tag{18}$$

where $x(t) = (x_1(t), \cdots, x_n(t))^T$, $\alpha = (\alpha_1, \cdots, \alpha_n)^T$, $A = (a_{ij})_{n \times n}$, $D = (d_{ij})_{n \times n}$, $\delta(u) = (\delta_1(u), \cdots \delta_n(u))^T$. Suppose $\int | \delta_i(u) |^2 \; \Pi(du) < \infty, i = 1, \cdots, n$. It is straightforward to see that neither the linear growth condition nor the global Lipschitz condition will be satisfied by this system. However, system (18) the functions $\mu(x), \sigma(x)$ and $c(x,u)$ satisfy condition (i), In this case define $V(x) = \sum_{i=1}^n m_i \alpha_i h(x_i / \alpha)$. where $h(s) = s - 1 - \ln(s)$, for n positive constants $m_1, \cdots, m_n$. This function satisfies conditions (ii) and (iv) of, whilst it is straightforward to show that

$$LV(x) = -\frac{1}{2}(x(t) - \alpha)^T H(x(t) - \alpha) + G(x), \text{where } H = -CA - A^T C - D^T diag(c_1\alpha_1, \cdots, c_n\alpha_n)D,$$

$$G(x) = \sum_{i=1}^n m_i \int [\alpha_i x_i \delta_i(u) - \alpha_i \ln(1 + \alpha_i \delta_i(u)) - \delta_i(u)(x_i - \alpha_i)]\Pi(du),$$

$C = diag(m_1, \cdots, m_n)$. Thus, if the n positive constants $m_1, \cdots, m_n$ can be found such that the symmetric matrix H is non-negative definite and $G(x) \le K_1 + K_2 V(x)$, then it follows that

$LV(x) \le K_1 + K_2 V(x)$ and condition (iii) is satisfied. Therefore, the Euler scheme will converge to the true solution of (18) in the sense of Theorem 4, provided that the time step Δt is sufficiently small.

V. Conclusion

In this paper, we demonstrated that the Euler approximation solution converge to the true solution $x(t)$ if both remain within a compact set. A compact set appropriate is introduced; the escape times from this set, both of $x(t)$ and the approximate solution are bounded in probability. By these results, we prove our convergence result of Theorem 4, namely that, under certain conditions(conditions(i)-(iv)), weaker than Global Lipschitz and Linear growth conditions, the Euler scheme applied to system (1), converges to the analytic solution $x(t)$, and in doing so bound the order of this approximation. This method solve the numerical solutions which the linear growth and global Lipschitz conditions are not satisfied by systems.

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References

[1] R. Merton,"Option pricing when underlying stock returns are discontinuous," J. Fina. Econ.,**3** (1976)pp.125-144.
[2] M. Jeanblanc-Picque, M. Pontier, "Optimal portfolio for a small investor in a market model with discontinuous prices," Appl. Math.Optim., vol.22,1990,pp. 287-310.
[3] M. Sabrina, "An approximation of American option pricing in a jump-diffusion model," Stoc. Proc. Appl., Vol. 62,1996,pp. 1-17.
[4] Y. Maghsoodi, C. Harris, "In-probability approximation and simulation of nonlinear jump-diffusion stochastic differential equations," IMA J. Math. Contr. Info., Vol.4, 1987,pp. 65-92
[5] P. Kloeden, E. Platen,"Numerical solution of stochastic differential equations," Springef-Verlag: Berlin,1992, pp. 89-102.
[6] M. Federico, H. Thong,"On the functional estimation of jump-diffusion models," J. Econ.,Vol.116,2003, 293-328.
[7] P.Protter, D.Talay, "The Euler scheme for Lévy Driven stochasticdifferential equations," Anna. Proba., Vol. 25,1997,393-423.
[8] X. Mao, G. Marion, E. Renshaw, "Convergence of the Euler scheme for a class of stochastic differential equations," J. Inte. Math., Vol. 144,2002,9-22.

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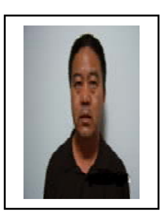

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