Super-Resolution Algorithm Based on SNR and Combination of Error-Parameter Analysis and Searching Method

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Abstract

Generally, super-resolution image restoration approach is an ill-posed problem and its inversion is stabilized by regularization. It is important to set proper regularization parameter. Furthermore in many practical situations the blurring process is generally unknown or is known only to within a set of parameters, it is necessary to incorporate the blur identification into the restoration procedure. Assuming geometric warp matrixes are known, this paper presents a scheme in which regularization parameter is estimated by Signal-Noise Ratio and error-parameter analysis is combined with searching method to identify blur function during super-resolution restoration. Simulation shows that in the scheme the method of estimating regularization parameter can improve adaptive control power, make convergence of restoration algorithm better during super-resolution image restoration, and adaptively adjust this parameter when low-resolution images have different Signal-Noise Ratio; the combination of error-parameter analysis and searching method can effectively identify blur function, which reduce the cost of computation and accelerate searching speed.
Keyword: Super-resolution, Regularization Parameter, SNR, Blur Identification, Error- parameter Analysis, Searching Method

I. Introduction

Super-resolution (SR) image restoration is one of the most spot-lighted research areas, because it can overcome the inherent resolution limitation of low-resolution (LR) imaging systems which can be still utilized and improve the performance of most digital image processing applications including medical imaging, satellite imaging and video applications etc..

Generally, SR image restoration approach is an ill-posed problem and its inversion is stabilized by regularization. Bose et al. [1] pointed to the important role of regularization parameter and a proposed CLS (constrained least square) SR reconstruction which generates the optimum value of the regularization parameter, using the L-curve method.

In most super-resolution restoration algorithms, blurring process is assumed to be known. In many practical situations, however, the blurring process is generally unknown or is known only to within a set of parameters, it is necessary to incorporate the blur identification into the restoration procedure. Wirawan et al. [2] proposed a blind multi-channel SR algorithm by using multiple finite impulse response filters. Nguyen et al. [3] proposed a technique for parameter blur identification and regularization based on GCV (generalized cross-validation) and Gauss quadrature theory.

Assuming geometric warp matrixes are known, this paper presents a scheme in which regularization parameter is estimated by Signal-Noise Ratio and error-parameter analysis is combined with searching method to identify blur function during super-resolution restoration. Simulation shows that the scheme can reconstruct super-resolution image and identify blur function effectively.

II. Mathematic Model

Super-resolution restoration can be presented as a sparse linear optimization problem. The implementation is operated by standard operations such as convolution, warping, sampling, and etc. The relationship between the LR images and the high-resolution (HR) image can be formulated as [4]

$$\mathbf{g}_k = \mathbf{D}\mathbf{H}_k^2\mathbf{F}_k\mathbf{H}_k^1\mathbf{x} + \mathbf{e}_k \qquad (2.1)$$

Considering only atmospheric blur, an alternate matrix formulation of the super-resolution problem is presented as

$$\mathbf{g}_k = \mathbf{D}\mathbf{F}_k\mathbf{H}_k^1\mathbf{x} + \mathbf{e}_k \qquad (2.2)$$

Considering only camera lens/CCD blur, another matrix formulation of the super-resolution problem is presented as

$$\mathbf{g}_k = \mathbf{D}\mathbf{H}_k^2\mathbf{F}_k\mathbf{x} + \mathbf{e}_k \qquad (2.3)$$

where $\mathbf{g}_k$ is a lexicographically ordered vector of the k-th LR image $\bar{g}_k$ with size $M_1 \times M_2$; $\mathbf{x}$ is a lexicographically ordered vector of HR image $\bar{x}$ with size $N_1 \times N_2$; $\mathbf{e}_k$ is a lexicographically ordered vector of noise with size $M_1 \times M_2$ which is generally seen to be the normally distributed additive noise; $\mathbf{F}_k$ is a geometric warp matrix of size $N_1N_2 \times N_1N_2$, $\mathbf{D}$ is the decimation matrix of size $M_1M_2 \times N_1N_2$, $\mathbf{H}_k^1$ is atmospheric blurring matrix of size $N_1N_2 \times N_1N_2$, $\mathbf{H}_k^2$ is camera lens/CCD blurring matrix of size $N_1N_2 \times N_1N_2$, $1 \le k \le K$ and K is the number of low-resolution images.

In conventional imaging systems (such as video cameras), camera lens/CCD (Charge Coupled Device) blur has more important effect than the atmospheric blur (which is very important for astronomical images). In this paper we use the model (2.3).

Stacking K vector equations from the different LR images into a single matrix-vector:

$$\begin{bmatrix} \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_K \end{bmatrix} = \begin{bmatrix} \mathbf{D}\mathbf{H}_1^2\mathbf{F}_1 \\ \vdots \\ \mathbf{D}\mathbf{H}_K^2\mathbf{F}_K \end{bmatrix}\mathbf{x} + \begin{bmatrix} \mathbf{e}_1 \\ \vdots \\ \mathbf{e}_K \end{bmatrix} = \begin{bmatrix} \mathbf{B}_1 \\ \vdots \\ \mathbf{B}_k \end{bmatrix}\mathbf{x} + \begin{bmatrix} \mathbf{e}_1 \\ \vdots \\ \mathbf{e}_K \end{bmatrix} \Leftrightarrow \mathbf{g} = \mathbf{B}\mathbf{x} + \mathbf{e} \qquad (2.4)$$

III. The choice of regularization parameter

As mentioned before, SR image restoration approach is an ill-posed problem because of an insufficient number of LR images and ill-conditioned blur operators. We must adopt regularization to stabilize the inversion of ill-posed problem. A priori knowledge concerning a desirable solution is represented by a smoothness constraint, suggesting that most images are naturally smooth with limited high-frequency activity. In most super-resolution algorithms, it is appropriate to introduce a regularization term $\|Cx\|^2$ about the amount of high-pass energy in the restored image. At the same time, the fidelity between observed LR images and simulated LR images is represented by $\sum_{k=1}^{K}\|g_k - B_kx\|^2$. A Lagrange multiplier or regularization parameter α is introduced to control the tradeoff between fidelity and smoothness. A cost function can be formulated as:

$$J(x)=\sum_{k=1}^{K}\|g_k-B_kx\|_{w_1}^2+\alpha\|Cx\|_{w_2}^2 \qquad (3.1)$$

where w_1, w_2 are weighted matrixes. In this paper, assuming geometric warp matrixes are known, regularization parameters are estimated by SNR (signal-to-noise ratio) [5]. Estimation steps are described as follow:

(1) observed LR images are interpolated in HR grid to obtain corresponding approximate HR images;

(2) calculate local variance of approximate HR images, and the maximum of local variance is seen to be image variance E, the minimum of local variance is seen to be noise ε;

(3) regularization parameters can be calculated by

$$\alpha_k=\frac{\varepsilon}{E},\ 1\le k\le K \qquad (3.2)$$

If all of observed LR images have the same SNR, regularization parameters α_k are same. When all of observed LR images have different SNR, regularization parameters α_k are different by the above steps. The estimation approach is simple and reduces the complexity of computation. When LR images have different SNR, the method can adaptively adjust regularization parameter during super-resolution restoration.

IV. Combination of error-parameter analysis and searching method

In some blind de-convolution algorithms, identification problem have been simplified by parametrizing blur function. With some knowledge of imaging system and environment, we can impose a blur degradation model with a few free parameters. Specially, Gaussian blur, defocus blur and motion blur can be modeled by a parameter.

Error-parameter analysis is first presented by mouyan zou [5], in which a error-parameter curve is created, and based on the curve the parameter of blur function is decided. The idea of the paper is to combine error-parameter analysis with searching method [6] to identify blur parameter, which can decrease the cost of computation and accelerate the searching speed. The implementation of the combination is followed:

1) determining blur type based on experience, and write parameter model of blur function;

2) selecting a searching range of blur parameter $[\min p \quad \max p]$ in which minimal parameter $\min p$ is searching start , and set a searching step β which is parameter precision;

3) first, setting $p=\min p+(i-1)\times\beta$;

where $1\le i\le(1+(\max p-\min p)/\beta)$

next, calculating blur function based on parameter model;

then, using certain super-resolution algorithm to get HR image x;

at last, evaluating the fidelity or restoration error

$E_i=\sum_{k=1}^{K}\|g_k-B_kx\|^2$;

estimating the parameter p_0 corresponding to minimal error, and set next searching range $[p_0-\beta \quad p_0+\beta]$;

4) reducing the searching step, return 2) and search again. Stop the search until searching step approaches desired parameter precision;

5) estimating the parameter p_0 corresponding to minimal error, get blur function based on parameter model, and output the result.

In this section, geometric warp matrixes are known which is same to the previous section.

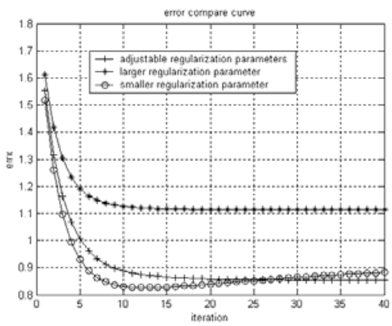
V. Simulation

A. *Estimation of regularization parameter*

In the simulation, an imaging model is adopted where no motions between LR images [7]. Different LR images correspond to different blur functions and have different SNR. The decimation is 2. Because CLS algorithm is sensitive to regularization parameter, this paper uses the algorithm to validate the method of estimating regularization parameter.

Because different LR images have different SNR, different regularization parameters corresponding to LR images are got using the method of estimating regularization parameter. The largest value of regularization parameters is used during the implementation of CLS and leads to a smoother solution in fig. 1(a); The smallest value of regularization parameters is used during the implement of CLS and leads to noise magnification in fig. 1(b), regularization parameters are adaptively adjusted during the implement of CLS and restored HR image is shown in fig. 1(c), Fig. 1(d) shows that the restored HR image in fig. 1(c) approaches true HR image best and the convergence of CLS algorithm is improved by adaptively adjusting regularization parameters.

(a) (b) (c)



(d)

Fig. 1. Restored HR image by (a) setting larger regularization parameter, (b) setting smaller regularization parameter, (c) adaptively adjusting regularization parameters, and (d) error compare among three kinds of the setting of regularization parameter

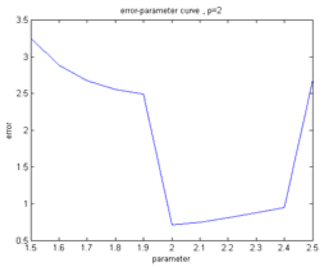


Fig. 2. When blur parameter is 2, the searching method is combined to error parameter analysis to identify blur parameter and Error-parameter curve is plotted. Error comparison is achieved by the fidelity $\sum_{k=1}^{K}\|g_k - B_k x\|^2$. The value of horizon coordination the lower point of the curve corresponds to is identified blur parameter.

B. Combination of error-parameter analysis and searching method

In the simulation the model of blur function is defocus one, the decimation is 4, image transform is rotation one and SNR is 30 db. 16 LR images are created based on imaging model (2.3). When parameter of defocus blur is 2, a larger searching range is set as [1 10], searching step is 1; next searching range is shrink to [1 3] using the combination of error-parameter analysis and searching method, searching step is adjusted to 0.5; last procedure is repeated, searching range is shrink to [1.5 2.5] and searching step is adjusted to 0.1 again. At last identified parameter is obtained. Identification result is shown in fig. 2.When parameter of defocus blur is 5, the same searching procedure is followed, and identification result is shown in fig. 3.

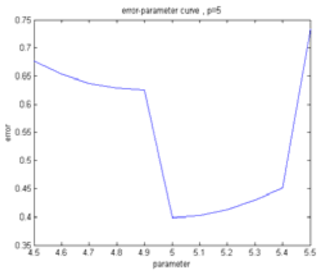


Fig. 3. When blur parameter is 5, the searching method is combined to error parameter analysis to identify blur parameter and Error-parameter curve is plotted. Error comparison is achieved by the fidelity $\sum_{k=1}^{K} \|g_k - B_k x\|^2$. The value of horizon coordination the lower point of the curve corresponds to is identified blur parameter.

VI. Conclusion

Assuming geometric warp matrixes are known, this paper uses CLS algorithm to restore HR image from a series of LR images, in which regularization parameters are estimated by calculating SNR of approximate HR images corresponding to different LR images. Because identification problem can be simplified by parametrizing blur function, this paper combines error-parameter analysis with searching method to identify blur parameter. The method of estimating regularization parameter can improve adaptive control power, make convergence of restoration algorithm better during image restoration, and adaptively adjust this parameter when low-resolution images have different SNR; the combination of error-parameter analysis and searching method can effectively identify blur function, which reduce the cost of computation and accelerate searching speed.

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