# Local Fuzzy Entropy-based Transition Region Extraction and Thresholding

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## Abstract

In this paper, we present a local fuzzy entropy-based transition method (LFE-TREM) and thresholding to improve the accuracy of local entropy based transition region extraction method (LE-TREM) and the performance of LE-TREM based thresholding, which effectively reduces the affects of Gaussian noise. Simulation results show that transition region extraction accuracy of LFE-TREM algorithm significantly outperforms the LE-TREM algorithm, and the quality of LFE-TREM based segmentation is better than that of LE-TREM algorithm for the image with White Gaussian noise.
**Keyword**: Image segmentation; transition region; local fuzzy entropy

## 1. Introduction

Image segmentation is one of the most difficult tasks in image processing and computer vision. The segmentation accuracy will greatly affect the latter feature extraction and target recognition tasks in computerized image analysis and understanding[1][2]. Thresholding is a fundamental approach to segmentation that enjoys a significant degree of popularity, especially in applications where speed is an important factor. The threshold values will determine the quality of segmentation. For this reason, considerable care should be taken to improve the probability of rugged segmentation.

Transition region based thresholding is a novel segmentation method developed in recent years[3−6]. Gerbrands[3] demonstrated the existence of transition region in image for the first time. Zhang and Gerbrands[4] introduced the transition region into image segmentation, and applied the effective average gradient (EAG) and clip transformation in their method. The main disadvantage of gradient-based EAG method is that much sensitive to noise. In order to limit the affects of noise, Liang and Le[5] modified the gradient operator with Gaussian weight. However, G-TREM is sensitive to salt & pepper noise. Yan and Sang [6] proposed local entropy-based transition method (LE-TREM). This method works well in image with salt & pepper noise. But for image with Gaussian noise, it couldn’t extract the transition regions effectively. In this paper, by analyzing the difference between transition regions and target regions (or background regions) we present a novel local fuzzy entropy-based transition region extraction method (LFE-TREM). Comparing experiments demonstrate that it is robust and effective to noisy image and significantly outperforms the LE-TREM.

## 2. Local entropy-based transition region extraction method

### *2.1 The main properties of transition regions*

Gerbrands[1] demonstrated transition regions universally exist in image and locate between the object and the background. Transition regions have the following properties:

1. Boundary characteristics: Transition region is the boundary between the object and the background, and cover around the object;
2. Area characteristics: In real image, even around the step edge there will be several pixels' width. So transition regions have certain width, and their area is not equal to zero;
3. The grayscale in transition region changes frequently: The frequent changes of grayscale bring abundant information to transition regions.

Gradient is good for sudden grayscale changes, but not the best measure for grayscale changes in TREM. As there are noises in image, the gradient is sensitive to noise. In noisy image, it is difficult to classify a pixel into transition regions or non-transition regions by its gradient. Local entropy can effectively describe the properties of the transition region.

### *2.2 Local entropy-based transition region extraction method*

Local entropy-based transition region extraction method (LE-TREM) gives up using gradient operators to calculate the gradient of image and determining transition region according to the peaks of clip curves. It extracts transition region by the local entropy of the image. The local entropy is defined as:

$$E(\Omega_k) = -\sum_{j=0}^{L-1} P_j \log P_j \,, \tag{1}$$

where $\Omega_k$ represents a small neighborhood with size $M_k \times N_k$ in image.

$$P_j = \frac{n_j}{M_k \times N_k} \,, \tag{2}$$

$P_j$ is the probability of grayscale j appears and $n_j$ is the number of pixels with grayscale $j$ in the neighborhood.

We can get an image about local entropy. Using a threshold, it is would be clearly known that the pixels belong to transition region or not. The threshold is defined as:

$$E_T = \alpha E(\Omega_k)_{\max} \,, \tag{3}$$

$E(\Omega_k)_{\max}$ is the maximum value in the local entropy image and coefficient $\alpha \in (0,1)$ is chosen with experience. LE-TREM effectively eliminates the influence caused by salt & pepper noise.

Without Gaussian noise, the transition region that extracted by this method would be complete and precise. LE-TREM essentially distinguished the gray-level kinds in the local region. When there are high degree Gaussian noises in the image, a small neighborhood in the target region and background region might have plenty of gray-level. It would increase the local entropy of this neighborhood. For extreme instance, local entropy would reach maximum value when the gray-levels different each other in the neighborhood within target (or background) region. Then the pixel must be divided into transition region though it belongs to target (or background) region. On the other side, as the effect of Gaussian, the pixels within transition region maybe be divided into non-transition region because that its neighborhood contains a few gray-levels. Local entropy can effectively describe the properties of the transition region, however, local fuzzy entropy can better describe the uncertain information of the transition regions in noisy image. Thus we introduce local fuzzy entropy in transition region extraction and thresholding.

# 3 Local fuzzy entropy-based transition region extraction method

### *3.1 The difference between transition regions and non-transition regions in noisy image*

The transition regions surround target regions and have a definite width. The change of gray-levels in transition regions is more frequent than in non-transition regions. For different noise kinds, the effect of noise would not be the same. The gradient within non-transition region will increase by the affection of salt & pepper noise. So G-TREM isn't suit for image with salt & pepper noise. The change frequency within non-transition region will reach a considerable high level by the affection of Gaussian noise. It makes LE-TREM to extract transition region difficultly. All of these methods cannot suit for image with salt & pepper noises or Gaussian noises.

Under the affection of salt & pepper noises (or Gaussian noises), the concentrated degree of non-transition regions has a little change. In other words, most of gray-levels within non-transition regions will focus on the gray-level average of the neighborhood. As the property of transition region, only few of gray-levels in transition regions distribute near the average of the neighborhood. Local fuzzy entropy can well distinguish this kind difference; even there are high degree Gaussian noises in the image.

### *3.2 Local fuzzy entropy*

According to the fuzzy theory, the belonging of an element to a fuzzy set is characterized by membership or belonging function. If *U* is a set of objects u, a fuzzy set *A* in *U* is defined by $A = \{(u, \mu_A(u)) | u \in U\}$. Where $\mu_A(u)$ is called the membership function of the fuzzy set *A*. The measure of the uncertainty associated with vagueness is employed to evaluate the degree of fuzziness.

An image with size $M \times N$ and L gray-level can be treat as fuzzy lattice according to fuzzy theory. Then the image *f* will be:

$$f = \begin{bmatrix} \mu_{11}(f(1,1)) & \mu_{12}(f(1,2)) & \cdots\cdots & \mu_{1N}(f(1,N)) \\ \mu_{12}(f(1,2)) & \mu_{22}(f(2,2)) & \cdots & \mu_{2N}(f(2,N)) \\ \cdots & \cdots & & \cdots \\ \mu_{M1}(f(M,1)) & \mu_{M2}(f(M,2)) & \cdots\cdots & \mu_{MN}(f(M,N)) \end{bmatrix}, \tag{4}$$

$\mu_{MN}(f(M,N))$ is the single dot membership of the fuzzy lattice and its coordinate is （M，N）. In other words, the degree of some character at the （$M$，$N$）pixel is $\mu_{MN}$ （$0 \leqslant \mu_{MN} \leqslant 1$）.

Membership function may be calculated by the standard S function, and standard *S* function is defined as:

$$S(f(m,n);a,b,c)=\begin{cases}0; & f(m,n)\le a\\ 2((f(m,n)-a)/(c-a))^2; & a< f(m,n)\le b\\ 1-2((f(m,n)-c)/(c-a))^2; & b< f(m,n)< c\\ 1; & f(m,n)\ge c\end{cases}, \tag{5}$$

in (5), $b=(a+c)/2$. The degree of ambiguity in an image can be measured by the entropy of fuzzy set A:

$$H(A)=\frac{1}{n\ln 2}\sum_{n} S_n(\mu_A(X_i)),\quad i=1,2,\cdots,n\ , \tag{6}$$

where $S_n(\cdot)$ is the Shannon's function: $S_n(x)=-x\ln x-(1-x)\ln(1-x)$. If the concept of fuzzy entropy is used in two-dimension image, the fuzzy entropy of image would be:

$$H(f(m,n))=\frac{1}{MN\ln 2}\sum_{m}\sum_{n} S_n(\mu_{mn}(f(m,n)))\ , \tag{7}$$

It is easy to extend the fuzzy entropy to local fuzzy entropy when the fuzzy entropy of a small neighborhood is calculated in image. From (5) and (7), some important conclusions would be:

1. The non-transition regions have higher local fuzzy entropy because that their gray levels are distributed concentratedly;
2. The transition regions have lower local fuzzy entropy because that their gray levels are distributed dispersively;
3. The transition regions can be extracted from an image according to this kind of difference.

### *3.3 Transition regions extraction and thresholding based on local fuzzy entropy*

Firstly, the size of neighborhood $\Omega_k$ is important for LFE-TREM. If the size is smaller, as there are fewer pixels within the neighborhood, the local fuzzy entropy cannot measure the fuzzy property exactly. On the other side, if the size is bigger, the location will be vague. Experiments show that the sizes from 7×7 to 15×15 are suit for LFE-TREM. After the size of neighborhood $\Omega_k$ has been fixed, the membership of the pixel's gray-level within neighborhood is calculated by S function. But the S function must be modified to fit LFE-TREM:

1. in (5), $b=\frac{1}{M\times N}\sum_{m}\sum_{n} f(m,n)$, $m=1,2,\cdots,m$, $n=1,2,\cdots,n$, M × N is the size of neighborhood and $f(m,n)$ is the pixel's gray-level within neighborhood;
2. the fuzzy bandwidth $W$ is the bigger one of $f_{\max}-b$ and $b-f_{\min}$ : $W=Max((f_{\max}-b),(b-f_{\min}))$ ， $f_{\max}$ is the maximum gray-level and $f_{\min}$ is the minimum gray-level within neighborhood;
3. in order to fit LFE-TREM, $a=b-W$ and $c=b+W$ in (5).

When the membership of each pixel within neighborhood has been calculated, we can get the local fuzzy entropy $H(\Omega_k)$ by (7). After the neighborhood window scans the whole image from left to right, and from top to bottom, a local fuzzy entropy image (LFEI) will be found. It is made up of the local fuzzy entropy of each pixel. The pixels within transition region will have smaller values than that within non-transition regions in LFEI.

Secondly, $H_T=\beta H(\Omega_k)_{\max}$ will be used to segment the local fuzzy entropy image. $H(\Omega_k)_{\max}$ is the maximum of local fuzzy entropy. $\beta\in$（0，1）is according to the quality of image. The extracted parts by $H(\Omega_k)<H_T$ are the transition regions.

Finally, the whole image is segmented through transition regions. There are many ways to segment the whole image by histogram of extracted transition regions[7]. The threshold could be mean arithmetical value of pixel's gray-level within transition regions or maximum of transition region's histogram. In this paper, the latter is taken and the segmented image is deal with using median filter.

## 4 Simulation and performance of segmentation

The experiment is performed on test image bacteria.tiff. The results are shown in Fig.1.
From the contrast between Fig.1(d) and (g), transition regions extracted by LE-TREM spread nearly the whole image. It can't represent the right transition regions, and this affect the performance of segmentation result. But favorable result can be get by LFE-TREM because that it can extract transition regions effectively for image with White Gaussian noise. The results show in Fig.1(i).

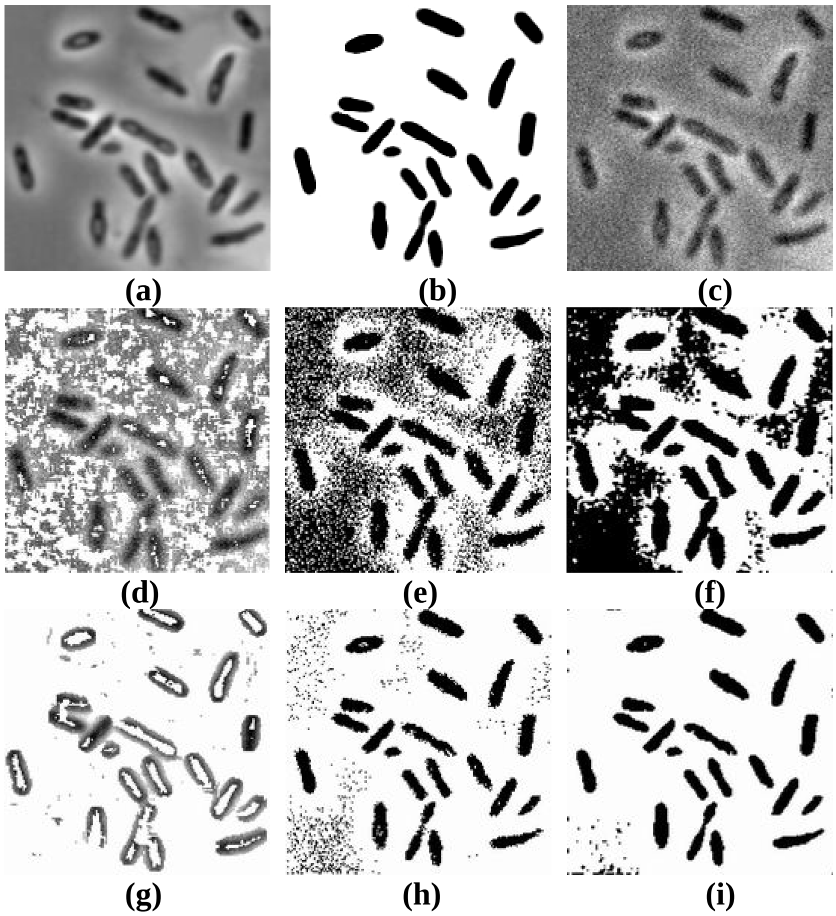


Fig.1. Transition region extraction and thresholding of noisy image by LE-TREM and LFE-TREM
(a) original bacteria image
(b) ground truth image
(c) bacteria image with White Gaussian noise ( $\sigma$ =20)
(d) transition region extracted of (c) by LE-TREM
(e) segmentation result based on (d)
(f) median filter of (e)
(g) transition region extracted of (c) by LFE-TREM
(h) segmentation result based on (g)
(i) median filter of (h)

For many years, lots of scholars proposed diverse methods for segmentation evaluation[8]. In these methods, misclassified error (*ME*) is fit in with evaluating the performance of segmentation[9]. *ME* is defined:

$$ME = MP / AP \tag{8}$$

in (8), *MP* is sum of the target pixels misclassified background pixels and the background pixels misclassified target pixels. *AP* is the sum of target pixels. The number of target pixels is calculated by ground truth image. We know that the lower ME value, the better the performance of segmentation. To cope with the random property of noise, five sample images are generated separately for each noise level. The final ME value will be their average.

In Fig.2, the performance curves of LE-TREM is marked with “○”. At the same time, the performance curves of LFE-TREM is marked with “●”. Fig2.(a) shows that the performance of LFE-TREM is close to that of LE-TREM in salt & pepper noisy image. Even the worst situation (0.20 Salt & pepper noise), the difference of these two methods is about 0.1s. In Fig.2(b), the evaluation curve of LFE-TREM is lower than that of LE-TREM, thus the performance of LFE-TREM is better than that of LE-TREM.

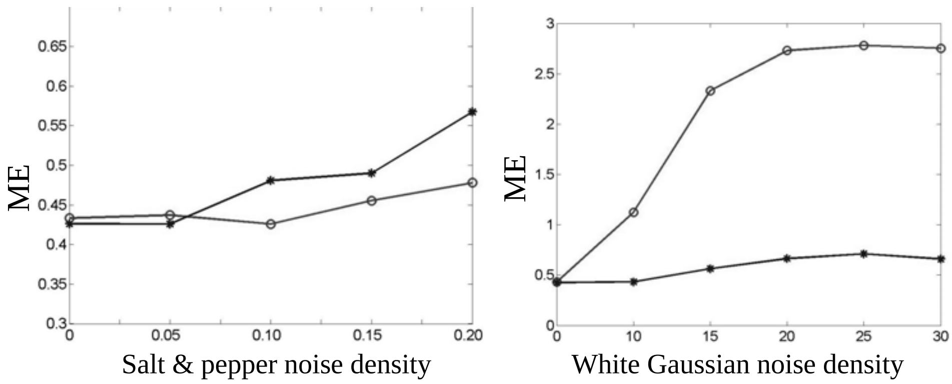


Fig.2. (a)performance curves of LE-TREM and LFE-TREM in salt & pepper noisy image (b)performance curves of LE-TREM and LFE-TREM in White Gaussian noisy image

## 5 Conclusions

LE-TREM is sensitive to White Gaussian noise and it will affect the performance of transition regions extraction. In seriously situation, it might result in an incorrect extraction of transition region and bad segmentation under White Gaussian noise. A novel local fuzzy entropy method (LFE-TREM) for transition region extraction and thresholding is proposed in this paper. By taking advantage of the difference of between transition regions and non-transition regions in noisy image, this method can extract transition regions accurately. Further more, the threshold of whole image is fixed through the right extraction of transition region. Simulation results show that LFE-TREM has higher robustness than LE-TREM in noisy image. The extraction of transition region by LFE-TREM is correct and the performance of segmentation is satisfied.

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