An Integrated Planning/scheduling Model for Multi-stage Manufacturing System

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Abstract

An integrated planning/scheduling model for multi-stage manufacture system was provided first. Then, based on material flow and inventory management to optimize the flow velocity and flow potential, and following decomposition approach, a feasible optimal production plan and schedule for the detailed job shop was developed. This model is expected to form better and feasible product plans, manufacturing sequences, workshop schemes, inventory programs and purchase schemes combined with MRPⅡand ERP
Keyword: Multi-stage manufacture system, Integrated planning/scheduling model, Decomposition approach.

I. Introduction

Traditional product planning procedures, e.g., those used in MRP (Material Requirement Planning) systems, MRPⅡ (Manufacture Resources Planning) and ERP systems follow a top-down hierarchical approach. As Bitran and Tirupati pointed out in 1989[1], they start with the generation of specific planned order released for all final products, subassemblies and components produced. Based on the order, they can get a set of tasks and due dates then. Thus, a detailed job shop scheduling problem is to find an operation sequence which satisfies these due date. Since the production planning procedure used in MRP, MRPⅡor ERP ignore detailed job shop scheduling constraints, there is no guarantee that a feasible production scheduling exists for the generated production plan.

In this paper, we build an integrated model for a multistage manufacture system planning and scheduling. Huang H J. gave a research on multistage manufacturing system planning, but he used the top-down hierarchical approach to solve the problem and the constraints of each workshop are imprecise. He also ignored the detailed workshop scheduling [2][3]. Here we build a planning model, which can be solved by space-time expanded network approach, and a detailed job shop scheduling model. Following decomposition approach, our solution method alternates between solving a planning and a job shop scheduling problem (Suppose that each manufacturing stage only contains one machine.). The procedure converges to a local optimum and can be terminated at any time with a feasible plan, which means that it allows a feasible schedule.

The paper organized as follows. In Ⅰ we present an integrated job shop planning and scheduling model. In Ⅱ and Ⅲ,we describe our procedure and its convergence. Part Ⅳ is the conclusion..

II. The Integrated Model for Multistage Manufacture System Planning and Scheduling

We now present an integrated planning/scheduling model, which can be described as two parts: one is planning model, the other is scheduling model.

A. Planning Model

We consider a production planning model with *M* products over a planning period horizon of *T* periods. Each planning period consists of *c* time units. This manufacture system has only one terminate operation and *N* stages. Denote $g_j[x_j(t)]$ as *j* station's capacity in period *t*, it was assumed to be an always increasing and differential concave function . In this paper, we let $g_j[x_j(t)=s_j+s_j'[1-exp(-k_jx_j)]$, where s_j is normal capacity of *j*; s_j' is the max variable capacity of *j*; k_j is the responding coefficient of variable capacity to lotsize; let $x_j[t]=\sum_m b_j^m x_j^m(t)$, b_j^m is the weight coefficient, through this equation we can translate the different product into standard product.

In this model, the target function is the system's cost function. The model *P(X,I)* can be formulated as follows:

$$P(X,I)\begin{cases} \min Z=\sum_t\sum_m\left\{\sum_j\left[p_j^m x_j^m(t)+I_j^m(t)c_j^m\right]+x_d^m(t)f^m\right\} \\ s.t. \\ x_j^m(t)=I_j^m(t-1)+x_{j-1}^m(t)+F_j^m(t)-I_j^m(t) \\ x_N^m(t)=x_d^m(t) \\ \sum_m b_j^m x_j^m(t)\le g_j\left[x_j(t)\right] \\ I_j^m(0)=I_j^m \\ x_j^m(t), I_j^m(t)\ge 0; m\in M;\ j\in N; t=1,2,...,T \end{cases} \tag{1}$$

Where:

$x^m_j(t)$: the volume of product *m* on machine *j* at the end of period *t*
$x^m_d(t)$: the volume of product *m* at the virtual node *d* at the end of period *t*
$F^m_j(t)$: the volume of product *m* come from environment and need performing on machine *j* at the end of period *t*
$I^m_j(t)$: the inventory level of product *m* on machine *j* at the end of period *t*
I^m_j: the inventory level of product *m* on machine *j* at the beginning
p^m_j : per unit product *m* cost performed on machine *j*
$c^m_j(t)$: the hoding cost of product *m* on machine *j*
$f^m(t)$: the fine function. It can be expressed as: if $\delta^m \geqslant t*c$, $f^m(t)=(\delta^m-t*c)K_L$, and if $\delta^m \leqslant t*c$, $f^m(t)=(t*c-\delta^m)K_E$ .here δ^m is *the* due date of product in period *t*, K_E, and K_L stand for respectively advance punish coefficient, delay punish coefficient.

B. *The Scheduling Model*

We consider the scheduling model with N products over T periods. Each planning period consists of c time units, so we can know that the model deals with the operation sequence on each machine during c time units.

Since we suppose that each stage only includes one machine, each period production jo can be looked as N products performed on M machines in a job shop problem. Thus, we can use job shop scheduling method to solve the multistaged manufacture system's scheduling problem.

Whenever $x_d^m(t)>0$ for some product and period t, this product quantity represents a production job to be performed in a single job shop with M machines. Because each stage only have one machine. Let O_{jmt} denote the operation to be performed on machine j for job (m, t), so the scheduling problem translate into find a operation sequence { O_{jmt} : m=1, 2, …, T} for any given pair {(j, t) : j=1, 2, …, N}. Let the (zero-one) vector *y* specify the sequence. Feasible start time of all operations can, for a given vector *y*, be specified as the feasible times of the nodes in a PERT network *(N(y), A(y))*, which of course depends on the given sequence y. *N(y)* is the set of nodes and *A(y)* is the set of arcs between operations.

The costs are given by a function *g(X, I)* of the production quantity vector $X=[x^m(t)$: m=1, 2, …, M; t=1, 2, …, T], and the inventory level vector $I=[I^m(t)$: m=1, 2, …, M; t=1, 2, …, T], thus *g(X, I)* could be express as :

$$g[X,I]=\sum_{m,t}[c_j^m I_j^m(t)+p_j^m x_j^m(t)]+\sum_j[c-\sum\tau_j^m x_j^m(t)]*w_j \tag{2}$$

j=1, 2, …, N; m=1, 2, …, M; t=1, 2, …, T

where $c^m_j(\geqslant 0)$ is backlogging cost of per unit product *m* on machine *j*; τ^m_j a per unit processing time and w_j a per unit cost of machine *j*.

The scheduling model *G(X, I)* can be formulated as (3).

$$G(X,I)\begin{cases}\min g[X,I]=\min\sum_{m,t}[c_j^m I_j^m(t)+p_j^m x_j^m(t)]+\sum_j[c-\sum\tau_j^m x_j^m(t)]*w_j\\ s.t.\\ x_j^m(t)+I_j^m(t)=F_j^m(t)+x_{j-1}^m(t)+I_j^m(t-1)\\ \sum_m x_j^m(t)\tau_j^m\le c\\ t_N^m(t)+x_N^m(t)\tau_N^m(t)\le t*c\\ t_1^m(t)+\sum_j x_j^m(t)\tau_j^m\le t*c\\ I_j^m(0)=I_j^m\\ x_j^m(t)\ge 0\end{cases} \tag{3}$$

Where the τ^m_j, c^m_j, w_j, $F^m_j(t)$, $I^m_j(t)$, I^m_j, and $x^m_j(t)$ see above. j=1, 2, …, N; m=1, 2, …, M; t=1, 2, …, T; $t^m_j(t)$ is the start time of product *m* on machine *j* during the period *t*.

III. The Procedure

A. The Pprocedure of Planning Model

First, we follow spance-time expanded network approach to solve the planning model (see Huang HJ. 1998). i.e., figure1 represents the network of three stage system, N=3, T=5.

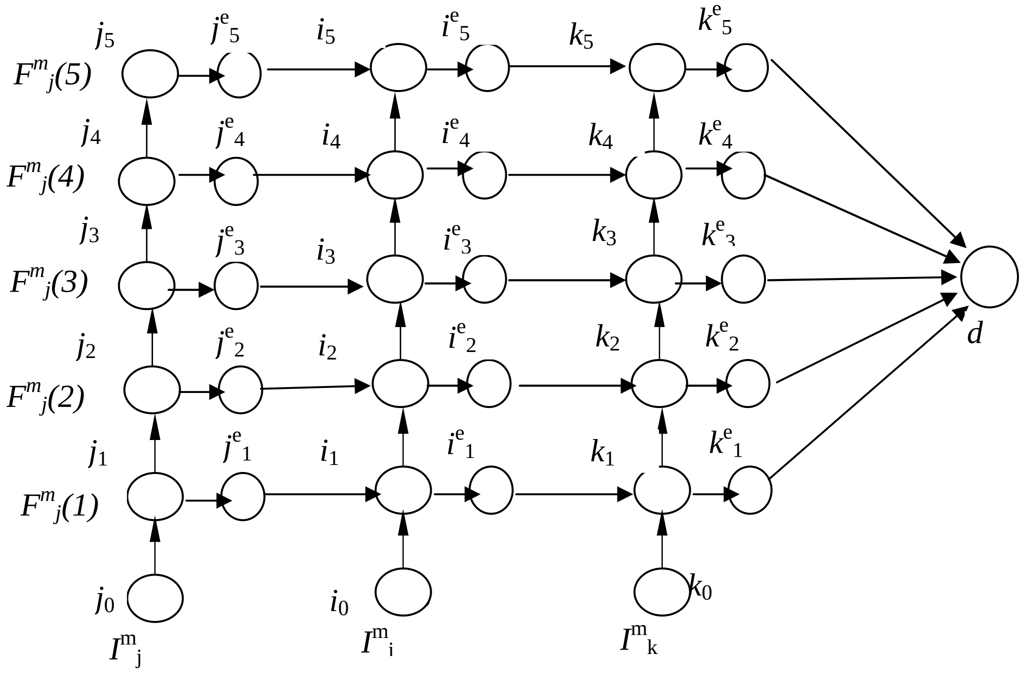

Figure1 the time-space extended network of M=3, T=5 manufacturing system

Each node j is expanded to $\{j_t$: t=0, 1, 2, …, T$\}$ nodes and $\{j^e_t$: t=0, 1, 2, …, T$\}$ nodes. Denote the set B of alll the arcs, we can get: $B=\{(j_t, j^e_t)$: t=0, 1, 2, …, T$\} \cup \{(j^e_t, j_t)$: t=0, 1, 2, …, T$\} \cup \{(j_{t-1}, j_{t1})$: t=0, 1, 2, …, T$\} \cup \{(j^e_t, j_{t+1})$: t=0, 1, 2, …, T$\} \cup \{(N^e_t, d)$: t=0, 1, 2, …, T$\}$

Let $F^m_j(t)$ flow in network at node j_t; I(0)flow in network at node j_0; arc (j_{t-1}, j_t) re presents $I^m_j(t)$ and the cost c^m_j; arc $(j^e_t, (j+1)_t)$ represents $x^m_j(t)$ and the weight is zero; arc (j^e_t, d) represents $x^m_N(t)$ and the weight is $f^m(t)$. where j=1, 2, …, N; m=1, 2, …, M; t=1, 2, …, T.

Now we have translated the dynamic planning problem into an equivalent static minimum cost network flow problem. After we add an item $\sum_{t=1}^{T}\sum_{j}\theta \max\left[0, \sum_{m} b_j^m x_j^m(t) - g_j\left(x_j(t)\right)\right]^2$ in the target function Z, where θ is a big enough positive number. Then we solve the problem by using Frank-Walfe algorithm as follows:

- ●*step1* initialization step, let vector U^m denote the arc's flow in expanded network. set $U^m=0$, *m*=1, 2, …, M
- ●*step2* calculate the arc unit under U^m , and find the shortest path of (j_t, d)
- ●*step3* distribute $F^m_j(t)$ and $I^m_j(t)$ to the shortest path , calculate the new arc's flow V^m
- ●*step4* set $U^m(\lambda)=U^m+\lambda(V^m-U^m)$, *m*=1, 2, …, M; $0\leqslant\lambda\leqslant 1$, λ is the length of iterative pace.
- ●*step5* if $Z(U^m)-Z(U^m(\lambda)) \leqslant \delta$, δ is a little positive number; then stop;
  else , let $U^m=U^m(\lambda)$, *m*=1, 2, …, M; go step 2

REMARK1: The convergence of the procedure has been validated by Huang HJ.[4][5], this model can obtain a local optimum $x^m{}_j\ (t)$ and $I^m{}_j(t)$, under the fixed condition (plan) $F^m{}_j\ (t)$ and $I^m{}_j$.

B. *The Multi-pass Procedure of Planning/Scheduling*

After we have the procedure of planning model, now we can use a decomposition approach and a multi-pass procedure to solve the integrated planning/scheduling model. The method alternates between solving the planning problem for a fixed choice of *y* and a job shop scheduling problem for a fixed choice *(X,I)*. The procedure is described as follows:

- *step1* get a initial sequence y_0 as follow: first solving *P(X,I)* under fixed *F* and I_0, we get the optimum (X^*, I^*);second, solving *G(y)* for fixed (X^*, I^*) and find the optimum y^*, let $y_0= y^*$
- *step2* set $k=0$, $g_{-1}=+\infty$, and list:=Φ
- *step3* solve $G(y_k)$, $g_k:=g[X(y_k),I(y_k)]$, if $g_k \leqslant g_{k-1}$, then set list:={ y_k }
- *step4* find a sequence $y' \notin$ list, subject to:

$$t_1^m(t)+\sum_j x_j^m(t)\tau_j^m \le t*c$$

if there is no such *y'*, then go to step 5
else set $k=k+1$; $y_k=y'$, and go to step 2

- *step5* define $TC(k)=Z(X(y_k),I(y_k))+C(y_k,X(y_k))-Z(X(y_{k-1}),I(y_{k-1}))-C(y_{k-1},X(y_{k-1}))$

if $TC(k) \leqslant \delta$, then stop
else
solving $P(X(y_k),I(y_k))$ under fixed $X(y_k)$ and $I(y_k)$, get the optimum $(F^*, I_0{}^*)$;
solving *P(X,I)* under fixed $(F^*, I_0{}^*)$, get the new optimum (X^*, I^*);
solving *G(y)* for the new (X^*, I^*) and find the optimum y^*

set $k=k+1$, $y_k= y^*$; go to step 2

REMARK2: Bacause *G(y)* and *P(X,I)* are both linear program, the solution to *G(y)* and *P(X,I)* is easy and fast.

REMARK3: It is easy to know that $\{g(X(y_k),I(y_k))\}_k$ is a nonincreasing sequence. Since in general, g{., .} is bound below ($g \geqslant 0$),the sequence is $\{g(x^k)\}_k$ converges to g^*.

According to J.B. Lasserre 1992, Adams J.1988 and Salomon M. 1991, this procedure will stops after a finit number og iterations with terminal sequence y^* and a production plan X^*. More over X^* is a local optimal solution of problem. So, we can know our procedure converges to local optimal solution X^* and y^*.

Ⅳ. Conclusion

In this paper, we have presented an integrated production planning and scheduling model for multi-staged manufacture system. Following decomposition approach, in contrast to top-down hierarchical approach, our solution generated production plan is feasible. Further more, the scheduling of detailed job shop base on this aggregate plan is also feasible. After further numeral experiment, the integrated model is expected to be used in MRP, MRPⅡand ERP system to improve their planning and scheduling quality.

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