An Intelligent Sliding-Mode Control Algorithm for Position Tracking Servo System

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Abstract

This paper mainly discusses a control algorithm based on the principles of the sliding mode variable structure control and fuzzy control for the position tracking servo system. The equivalent control is first determined using pole placement. Then, the reaching condition is guaranteed by the use of the robust feedback control with switching gains. Finally, fuzzy tuning schemes are then employed to accelerate the reaching phase and reduce chattering. The simulation results show that the high performance and attenuated chatter are achieved. The rapidness and robustness of the system are improved. Moreover, this control system realization is simple and convenient.
Keyword: Sliding mode, Variable structure control, Pole placement, Fuzzy control.

I. Introduction

In recent years, Variable structure control with sliding mode attracts control domain's attentions, and has been widely developed. The main drawback of sliding mode control (SMC) is "chattering " which can excite undesirable high-frequency dynamics. Several methods of chattering reduction have been reported. But many approaches provide no guarantee of convergence to the sliding mode and involve a tradeoff between chattering and robustness. Continuous SMC, as proposed in [1], can exponentially drive the system state to a chattering-free sliding mode but tends to produce conservative designs. Reduced chattering may be achieved without sacrificing robust performance by combining the attractive features of fuzzy control with SMC. Fuzzy logic is a potent tool for controlling ill-defined or parameter-variant plants. By generalizing fuzzy rules, a fuzzy logic controller can cope well with severe uncertainties. Fuzzy schemes with explicit expressions for tuning can avoid the heavy computational burden.

II. Control Design

Let a linear system be defined as

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + f(x,t) \tag{1}$$

where $x(t) \in R^n$ is the output, $u(t) \in R^m$ is the control input, and $f(x,t) \in R^n$ is the disturbances and unmodeled dynamics; $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are the nominal system constant matrices with $rank(B) = m < n$; ΔA and ΔB are uncertainties. The following assumptions are made

a) The uncertainties are continuous matrix functions of the uncertain parameters $p \in P \subset R^p$

$$\Delta A = \Delta A(p) \qquad \Delta B = \Delta B(p) \tag{2}$$

b) There exist matrices $D(p) \in R^{m\times p}$, $E(p) \in R^{m\times m}$, and $v(x,t) \in R^m$ such that the following matching conditions are satisfied

$$\Delta A = AD(p) \text{ and } \max_{1\le j\le m}\left|D_{ji}(p)\right| \le \delta_i,\ i = 1, 2, \cdots, n,\ \forall p \in P \tag{3}$$

$$\Delta B = BE(p) \text{ and } E(p) = diag(E_{jj}),\ \max_{1\le j\le m}\left|E_{jj}(p)\right| \le \varepsilon < 1,\quad \forall p \in P \tag{4}$$

$$f(x,t) = Bv(x,t) \text{ and } \left|v_j(x,t)\right| \le v_j,\quad \forall x, \forall t \qquad j = 1, 2, \cdots, m \tag{5}$$

In order to design the sliding mode variable structure controller, a switching function of dimension m is to be chosen $s(x) = Cx$, where $C = [C_1 \quad C_2 \quad \cdots \quad C_m]$ is an $m \times n$ constant matrix. The sliding mode is the constrained motion of the states along the trajectories on the sliding surface $s(x)$.

A. *Equivalent control*

For the system, $\dot{s}(x) = 0$ is a necessary condition for the state trajectory to remain on the sliding surface $s(x)$. When $\Delta A = 0$, $\Delta B = 0$ and $f(x,t) = 0$, if $C(Ax + Bu_E) = 0$, the state trajectory remains on the switching surface, where u_E is the equivalent control input. Linear feedback is proposed for u_E to assign the desired dynamics to the closed-loop system

$$u_E = -K_E x \tag{6}$$

where the equivalent control gain K_E can be obtained from a pole-placement technique, and K_E can be expressed as $K_E = (CB)^{-1}CA$, where CB is invertible. We can obtain the switching coefficient matrix C from the equation $(A - BK_E)^T C^T = 0$.

B. *Switching control*

The system trajectory under the condition that the state will move toward and reach the sliding surface is called the reaching mode or reaching phase.

Choosing Lyapunov function $V(x) = \frac{1}{2}s^2$, then, the following equation can guarantee the reaching condition be satisfied

$$\dot{V}(x) = s\dot{s} = sC\dot{x} < 0 \tag{7}$$

Considering the uncertainties in the system, the sliding mode controller can be designed as follows

$$u = u_E + u_S = -K_E x + u_S \tag{8}$$

and the switching control vector u_S is

$$u_{S,j} = -\left[\frac{v_j}{1-\varepsilon} + \sum_{i=1}^{n} \frac{\delta_i + \varepsilon\left|K_{E,ji}\right|}{1-\varepsilon}\left|x_i\right|\right]\mathrm{sgn}(\varphi_j(x)),\quad j = 1, 2, \cdots, m \tag{9}$$

where $K_{E,ji}$ is the element of the jth row and ith column of K_E.

$$\varphi_j(x) = s^T C b_j \tag{10}$$

and b_j is the *j*th column of B.

C. *Fuzzy control*

An additional control signal u_F is introduced to accelerate the reaching phase and to reduce chattering while maintaining sliding behavior. The fuzzy control term $u_{F,j}$ can be defined as

$$u_{F,j} = -\gamma_j \varphi_j(x) \qquad j = 1, 2, \cdots, m \tag{11}$$

where γ_j is a weighting factor. The component $u_{F,j}$ will continuously be adjusted by the use of fuzzy logic, depending on both s_j and $\dot{s}_j$ or the change of s_j and Δs_j [2]. We neglect the change of Δs_j. Using fuzzy labels *large* and *small* for $|s_j|$, the membership functions can be defined as following

$$\mu_{s_j - l\arg e} = 1 - \exp\left(-\frac{|s_j|}{\sigma_{s_j}}\right), \qquad \mu_{s_j - small} = \exp\left(-\frac{|s_j|}{\sigma_{s_j}}\right) \tag{12}$$

wher, σ_{s_j} is the positive constant.

Note that the fuzzy control $u_{F,j}$ may be more efficient if it is tuned according to $\varphi_j(x) = s^T C b_j$ as defined in (10). The following rules are proposed

1) if φ_j is positive large, then $u_{F,j}$ is negative large;
2) if φ_j is positive small, then $u_{F,j}$ is negative small;
3) if φ_j is negative large, then $u_{F,j}$ is positive large;
4) if φ_j is negative small, then $u_{F,j}$ is positive small;

We use the max-min defuzzification method for the fuzzy schemes above. Considering the case $\varphi_j > 0$ and choosing sigmoidal membership functions for φ_j, singletons for $u_{F,j}$, the following results can be obtained [3]

$$\gamma_j = \gamma_{m,j}\left(1 - \exp\left(-\frac{|s_j|}{\sigma_{s_s}}\right)\right) \tag{13}$$

where $\gamma_{m,j}$ and σ_{s_j} are tuning parameters.

In summary, the fuzzy control $u_{F,j}$ can be written as follows

$$u_{F,j} = -u_{F_{m,j}} \tanh\left(\frac{\varphi_j}{\sigma_\varphi}\right) \tag{14}$$

So the robust sliding mode controller is designed such that

$$u = u_E + u_S + u_F \tag{15}$$

and from above, we can obtain u_E, u_S, u_F, γ_j, φ_j; then the state vector $x(t)$ asymptotically converges to zero.

III. Application

A. ***Model of the system***

Considering a position tracking servo system, in this case, the system includes a casement, a reducer and an electromotor. The moment of inertia that is converted into the motor's axis is J =0.00657 Kgm^2 . The torque coefficient of the electromotor is C_M , and its counter electromotive force voltage coefficient is C_e . The resistance of the armature loop is R =3 Ω . The decelerating ratio is i =1670. The maximum of the Coulomb friction torque has been given. The maximum value of the load disturbance f_d is 9.8 Nm . The gain is K = 304. $N(A)$ is the backlash characteristic, and the transmission clearance is 2.5 mil . In addition, the fluctuation of the system's parameters is 10 percent. Fig. 1 shows the structure of the system.

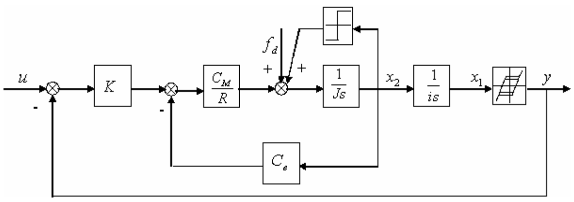

Fig. 1. Structure of the tracking servo system

The system's performances have been given. The transition time of the step response is t_s =1.6s. The overshoot is less than 17 percent. The steady-state error of unit step input is zero.

According to the system's requirement, we can neglect the backlash characteristic here. As shown in Fig. 1, suppose that the casement rotational angle is x_1, the angular velocity is x_2 , $y = x_1$, then the state equation of the control system is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0.0006 \\ 0 & -3.107 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 3778.79 \end{bmatrix} u + \begin{bmatrix} 0 \\ 152.207 \end{bmatrix} d \qquad (16)$$
$$y = [1 \quad 0]\, x$$

where u is the control input, and d is the total disturbance.

B. ***Design of the controller***

For the system above, in order to obtain zero steady state error of unit step response, the sliding mode should include the integral of the difference of the desired signal x_d and the casemate rotational angle x_1. Then assuming

$$\dot{x}_0 = x_d - x_1 \qquad (17)$$

The state equation of the system is constructed by Equations (16) and (17)

$$\begin{bmatrix} \dot{x}_0 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0.0006 \\ 0 & 0 & -3.107 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 377879 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 152207 \end{bmatrix} d + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_d \qquad (18)$$
$$y = [0 \quad 1 \quad 0]\, x$$

In the system, Equation (18) is controllable. In order to design the control law, we do the nonsingular linear transformation $x = Mz$, and obtain the following standard form

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -3.107 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 3778.79 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 152.207 \end{bmatrix} d + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} x_d \tag{19}$$

So in the new coordinate system, the sliding mode equation is

$$\begin{cases} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} z_3 \\ z_3 = -F \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \end{cases} \tag{20}$$

Considering the system's required properties, we determine the feedback coefficient matrix F = [0.00254 0.0062]. Consequently, in the new coordinate, we determine the switching function according to $\tilde{C} = (F, I_m)$ = [-0.00254 10.354 1]. The equivalent control gain can be obtained

$$u_E = -K_E x = -\tilde{K}_E M^{-1} x \tag{21}$$

with j =1, the switching control computed from (9) is

$$u_S = -\frac{v + \left(\sum_{i=1}^{3} \delta_i + \varepsilon |K_{Ei}| \right) |x_i|}{1 - \varepsilon} \operatorname{sgn}(s\hat{b}) \tag{22}$$

The fuzzy control u_F chosen according to (14) is

$$u_F = -u_{Fm} \tanh\left(\frac{\hat{b}s}{\sigma_t} \right) \tag{23}$$

where u_{Fm} and σ_t are some positive coefficients.

C. *Simulation*

Considering the position tracking servo system, with the desired eigenvalues chosen at {-4.8-0.64 j, -4.8+0.64 j}, the constants c_1 =-0.00254 and c_2 =10.354 are obtained. Assuming that the elements of the state equation (18) may fluctuate by 10% around their nominal values (ε =0.1), the bounds of uncertainties in the matching conditions can be determined as $\delta_i = 0.1\hat{a}_i / \hat{b}$, i =1,2,3. The bound v is determined by the maximal disturbance, v =0.0013. After some trials, the fuzzy tuning parameters are selected as u_{Fm} =0.1and σ_t =0.025.

According to the real time control requirement, under Matlab 6.0 environment, we adopt the fixed step-size arithmetic and simulate on the computer.Fig. 2 shows the tracking of a step input with fuzzy SMC (FSMC). Its transition time is 0.8s, and the overshoot is zero. Fig. 3 shows the response under the same condition using SMC. It is obviously that fast tracking and a significant reduction of chattering are obtained by introducing the fuzzy control component.

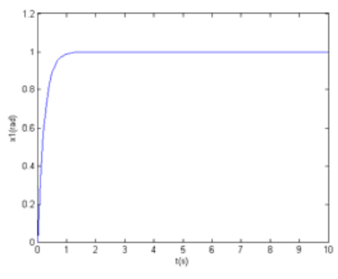

Fig. 2. Step response of FSMC

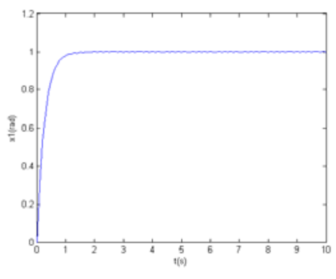

Fig. 3. Step response of SMC

Fig. 4 shows the ramp input response of the FSMC system. It shows that output can follow the input smoothly when the slope is half of the requirement (0.04rad/s). Fig. 5 shows the SMC system cannot follow smoothly with the input signal.

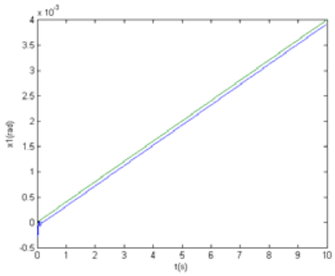

Fig. 4. Ramp response of FSMC

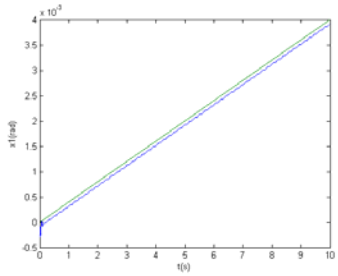

Fig. 5. Ramp response of SMC

IV. Conclusion

In this paper, a new technique combining the features of SMC and fuzzy control has been discussed. First, the equivalent control is determined from the desired sliding eigenstructure. The reaching condition is guaranteed by the use of the robust feedback control with switching gains. Fuzzy tuning schemes are then employed to accelerate the reaching phase and reduce chattering. The practical application of fuzzy logic proposed here is a computational intelligence approach to the engineering problems associated with explicit expressions. The computer simulation results show that the Fuzzy sliding mode control strategy improved the control performances of the system. The rapidness and robustness of the system are improved.

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