Application of Fractional Order Window For Sources Estimation

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Abstract

In this paper, a different approach to the problem of estimating the directions-of-arrivals (DOAs) of D sources by L sensors array (D < L) is presented. The main idea is the truncation of the received signal by a fractional order window whose parameters are used to reduce the DOA estimation error. The simulation results show that this approach offers a considerable improvement in the resolution of sources, as well as the estimations of their positions and their number and remedy to the problem of false estimation observed in the criteria Akaike Information Criteria (AIC) and Minimum Description Length (MDL) for unfavourable SNR.
Keyword: Fractional window, estimation, direction of arrivals, MUSIC.

I. Introduction

The automatic determination of the number of the reflective sources, their positions and other characteristic parameters from an appropriate processing of received data through an array of sensors find its application in different domains as radar/sonar and communication. Several algorithms of sources estimation have been developed. Schmidt and Bienvenu [1,2,3] have presented the MUSIC algorithm (MUltiple SIgnal Classification). To estimate the DOAs of narrow-band sources Ziskind [4], Wax [5] and Boehme [6] applied an algorithm of alternative projection based on the Maximum Likelihood Estimator approach (MLE). However the majority of these iterative procedures do not guaranties the convergence towards the global optimum. Furthermore, if the number of sources is large, these iterative procedures become very complicated. Some improvements in the performances of these algorithms have been made by several researchers. Wang in [7] proposed an active system for radar/sonar with a variation of the pulse frequency to improve the resolution in tone and the estimation of the DOAs of non-fluctuating sources. Huang and Barkat [8] applied the Frequency-Hopped technique for estimating the number of mobile sources by AIC and MDL criteria and they have found an improvements in the probability of false alarm, the probability of miss, and the probability of the correct detection compared to the monotony case. Barkat and Aissous [9] applied the Frequency-Hopped technique to improve the resolution of the DOA estimation using MLE. To estimate simultaneously the DOA and the multipath distribution delay, the TST-MUSIC algorithm (Time-Space-Time MUSIC) has been proposed in [10]. Kaushik [11] has proposed a new extension of the algorithm MUSIC to resolve the problem of spectral estimation.
In this paper, an approach of DOA estimation of sources, based on the use of a fractional order window, is presented. The problem's formulation is made in section 2. Some illustrations have been given in section 3 to show the performances of the proposed method. The conclusion is given in section 4.

II. Problem Formulation

Let's consider an array of L equidistant sensors and D narrow band sources centered in only one carrying frequency f, supposed to be distant enough from the array what makes possible to use the plane wave's model for the incidental signals. The response of the l^{th} sensor at the carrying frequency f can be expressed as follows:

$$x_l(m) = \sum_{k=1}^{D} \eta_{lk}(m).a_l(\theta_k).\exp(-j2\pi f \tau_{lk}) + n_l(m) \quad (1)$$

$$l=1,2,\ldots,L\ ;\ m=1,2,\ldots,M$$

where $n_l(m)$ is the component of the additive noise associated to the l^{th} sensor at the carrying frequency f. τ_{lk} is the delay between the k^{th} source and the l^{th} sensor. M is the number of samples. We propose in this work that the received signal is truncated using a pondered window of a fractional order defined by the following equation [12]:

$$W_F(m) = \frac{1}{2\pi}.\frac{\sin[(1-\alpha).\pi]}{\cosh\left[\left(m-\left(\frac{M-1}{2}\right)\right).\alpha\right] - \cos[(1-\alpha)\pi]} \quad (2)$$

$$0 < \alpha < 1, \qquad 1 \le m \le M$$

α is the fractional coefficient and M the number of samples.

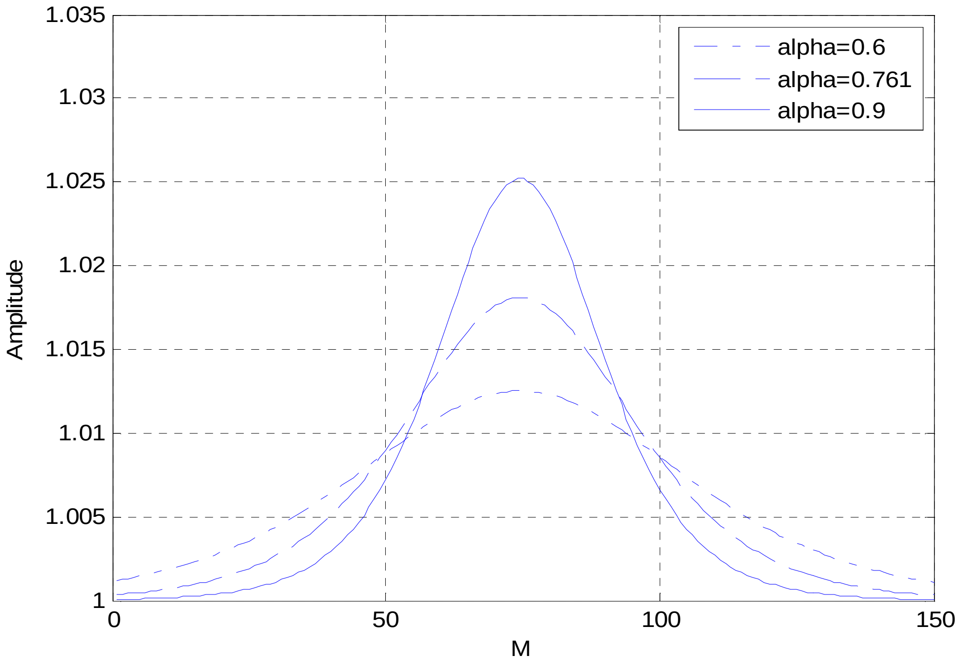

Figure (1): Plot of $W_F(m)$

This window was inspired by a distribution of the relaxation times of a fractional order system [13]. The truncated signal through the l^{th} sensor is given by :

$$X_{lF}(m) = \left[\sum_{k=1}^{D} \eta_{lk}(m).a_l(\theta_k).\exp(-j2\pi f \tau_{lk}) + n_l(m)\right] \times [W_F(m)] \quad (3)$$

By taking the first sensor as the origin of phase change, the delay, τ_{lk}, is given by:

$$\tau_{lk} = (l-k)(\Delta / c).\sin(\theta_k) \quad (4)$$

where c is the speed of propagation and Δ is the uniform spatial spacing between two adjacent sensors. $a_l(\theta_k)$ is the gain of the l^{th} sensor for an incidental signal following the direction θ_k

(k=1,2,…, D) at the carrying frequency f. The θ_k are the DOAs of the sources, η_{lk} is a random complex variable of the effect of the attenuations which is due to the propagation and the reflection on the k^{th} source and affecting the l^{th} sensor. Under the hypothesis that the sources are distant and the sensors are perfectly identical and omni-directional all the random complex variables η_{lk} are such that $\eta_{1k} = \eta_{2k} = \ldots = \eta_{Lk} = \eta_k$, where η_k, (k=1,2,…,D) are complex and Gaussian, and $a(\theta_k) = a_1(\theta_k) = a_2(\theta_k) = \cdots = a_L(\theta_k) = 1$. The observed data, at carrying frequency f, can be expressed in the matrix form, as:

$$X_F(m) = A_F(\theta).S(m) + N_F(m) \tag{5}$$

$S(m) = [\eta_1(m)\eta_2(m)\cdots\eta_D(m)]^T$ is the (Dx1) source vector, $X_F(m) = [X_{1F}(m)X_{2F}(m)\cdots X_{LF}(m)]^T$ is the (Lx1) observation vector and $N_F(m) = [n_1(m)n_2(m)\cdots n_L(m)]^T$ is the (Lx1) noise vector. $[.]^T$ indicate the transposed operation and $A_F(\theta)$ is an (LxD) matrix such as:

$$A_F(\theta) = \begin{bmatrix} \exp(-j2\pi f\tau_{11}) & \cdots & \exp(-j2\pi f\tau_{1D}) \\ \exp(-j2\pi f\tau_{21}) & \cdots & \exp(-j2\pi f\tau_{2D}) \\ \vdots & \cdots & \vdots \\ \exp(-j2\pi f\tau_{L1}) & \cdots & \exp(-j2\pi f\tau_{LD}) \end{bmatrix} \times [W_F(m)] = A(\theta) \times [W_F(m)] \tag{6}$$

where $A(\theta)$ is the (LxD) steering matrix and its columns $d(\theta_k)$ are the steering vectors. The analysis of $A(\theta)$ shows that each column, $d(\theta_k)$, corresponds to only one source and that all the sources have the same temporal carrying frequency f. Thus a mixture of D sinusoids having the same temporal carrying frequency f. Replacing τ_{lk} as given in equation (4) while the parameter Δ is taken to be equal to a half of the wavelength, the matrix $A(\theta)$ becomes:

$$A(\theta) = \begin{bmatrix} 1 & \cdots & 1 \\ \exp(-j\pi\sin\theta_1) & \cdots & \exp(-j\pi\sin\theta_D) \\ \vdots & \cdots & \vdots \\ \exp(-j\pi(L-1)\sin\theta_1) & \cdots & \exp(-j\pi(L-1)\sin\theta_D) \end{bmatrix} \tag{7.a}$$

$$A(\theta) = \begin{bmatrix} 1 & \cdots & 1 \\ \exp(-j.w_1) & \cdots & \exp(-j.w_D) \\ \vdots & \cdots & \vdots \\ \exp(-j(L-1).w_1) & \cdots & \exp(-j(L-1).w_D) \end{bmatrix} \tag{7.b}$$

The matrix $A(\theta)$ given by equation (7.b) becomes then a mixture of D signals with distinct spatial frequencies w_k ($w_k = \pi.\sin(\theta_k)$, k=1,…,D). Noting that each spatial frequency w_k contains the necessary information of the direction of arrival (DOA) of the corresponding source.

It was shown in [12] that the fractional window has the ability to separate a very close time frequencies of mixed sinusoids. Then the idea of using this fractional window in the preprocessing part is to enhance the separation of the spatial frequencies w_k contained in the mixed D signals.

The following hypotheses are supposed:

(i) For the carrying frequency f, M statistically independent and uncorrelated samples are taken.

(ii) The correlation matrix Rs of the vector source S(m) is defined positive.

(iii) The number D of the sources is lower than the number of sensors L.

(iv) The Noise $N_F(m)$ is Gaussian random process of zero mean and variance σ_n^2, stationary, ergodic and uncorrelated with the signals S(m).

The (LxL) correlation matrix of the observed data of equation (5) is obtained by multiplying it by its transpose conjugate complex and by taking its expectation as:

$$R_{XF} = A_F(\theta).R_S.A_F^H(\theta) + \sigma_n^2.I \tag{8}$$

Where R_s is the (DxD) matrix of correlation of sources and $\sigma_n^2.I$ is the (LxL) matrix of correlation of noise vector. Under hypothesis (i), the matrix R_{xF} can be estimated as follows:

$$\hat{R}_{XF} = \frac{1}{M}.\sum_{m=1}^{M} X_F(m).X_F^H(m) \tag{9}$$

To show the effect of fractional window on the quality of sources estimation the MUSIC algorithm and the AIC and MDL criteria have been used. The MUSIC algorithm [1,2] is based on the exploitation of the eigen-structure of the correlation matrix R_{xF}.. It is one of the most used algorithms in estimation since it has the advantages of being simultaneously able to estimate the number and the DOAs of the sources and very simple to implement. However it is less precise compared to other more complex algorithms such as the algorithms based on the maximum likelihood estimator (MLE) and other varieties of MUSIC algorithm [14]. Several works have been done to improve its performances [11,15]. In MUSIC algorithm the orthogonality measure function $P(\theta)$ is given as:

$$P(\theta) = \frac{1}{d^H(\theta).U_N.U_N^H.d(\theta)} \tag{10}$$

Under the constraint that the number of samples M is finite, the plot of the function P (θ) versus θ shows some peaks where their number is equal to the number of sources and their abscissas correspond to the DOAs of the sources.

Criteria AIC and MDL [16,17] are based on the information theory and they are very powerful techniques for estimating the number of sources. For the set of M observations, the number of sources is the value of the coefficient k, k ϵ {0,1,…,L-1}, which minimizes one or both of the two following equations:

$$AIC(k) = -2.\log\left[\frac{\prod_{i=k+1}^{L} \lambda_i^{\frac{1}{L-k}}}{\frac{1}{L-k}\prod_{i=k+1}^{L} \lambda_i}\right]^{(L-k).M} + 2.k \tag{11}$$

$$MDL(k) = -\log\left[\frac{\prod_{i=k+1}^{L} \lambda_i^{\frac{1}{L-k}}}{\frac{1}{L-k}\prod_{i=k+1}^{L} \lambda_i}\right]^{(L-k).M} + \frac{1}{2}.k.(2L-k).\log(M) \tag{12}$$

where λ_i are the eigenvalues of the correlation matrix.

III. Simulations and results

First of all the RMSE of the DOA estimation was used to set the best parameters for the fractional order window that will be used for the rest of the simulations. The second part of the simulations was concerned with the analysis of the performances of the proposed method in terms of the precision of the estimation, the field of vision of the algorithm and the angle of separation between the sources.

The Root Mean Square Error (RMSE) of the DOA estimation is defined by:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{Run}\left(\theta_k - \hat{\theta}_k\right)^2}{Run}} \tag{13}$$

where θ_k is the exact DOA and $\hat{\theta}_k$ is the estimated DOA of the k^{th} source respectively. Run is the number of executions of the calculation algorithm.

As a numerical example we consider 3 independent sources localized at 13°, 26° and 55° and supposed to be points far away at the same distance from a rectilinear array of 8 sensors. The sensors are omnidirectional, identical with unity gain and uniformly spaced of a halfwavelength $\Delta=\lambda/2$, where λ is the corresponding wavelength for the carrying frequency f.
The η_k, (k=1,2,..,D) are all taken identical Gaussian random variables with zero mean and variance δ thus the Signal to Noise Ratio, SNR_k (k=1,2,…,D) is defined as :

$$SNR_k = 10.\log_{10}\left[\frac{\mathrm{variance}(\eta_k)}{\sigma_n^2}\right] = 10.\log_{10}\left[\frac{\delta}{\sigma_n^2}\right] = SNR \qquad (14)$$

Figure (2) shows some plots of the RMSE of the DOA estimation as function of the SNR with the coefficient α as a parameter that is varied in the interval (0,1) with a very reduced step. From this figure, it can easily be seen that for a given SNR there exist some values of the coefficient α that can reduce the RMSE of the DOA estimation compared to RMSE of the DOA estimation without fractional order window. It is also found, for this example, that the best fractional window is obtained for the coefficient α =0.9. Hence, the fractional order window used for the rest of the simulations will be the one with the coefficient α =0.9.

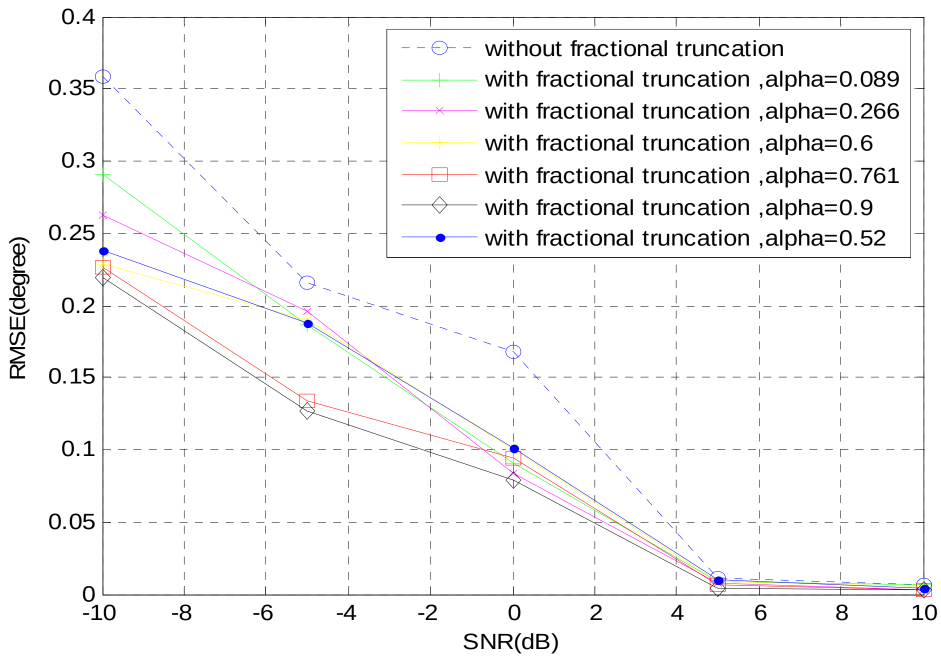

Figure (2): RMSE of the DOA estimation versus SNR for different values of the fractional coefficient α with L=8, M=150 and Run=150.

Figures (3) and (4) show the orthogonality function, given by equation (10), versus the DOA for an SNR=-5dB without and with the fractional window respectively. It can clearly be seen that the peaks of the orthogonality function in figure (4) are narrower and sharper than the ones in figure (3), therefore the precision in DOA estimation has been improved by using the fractional window.

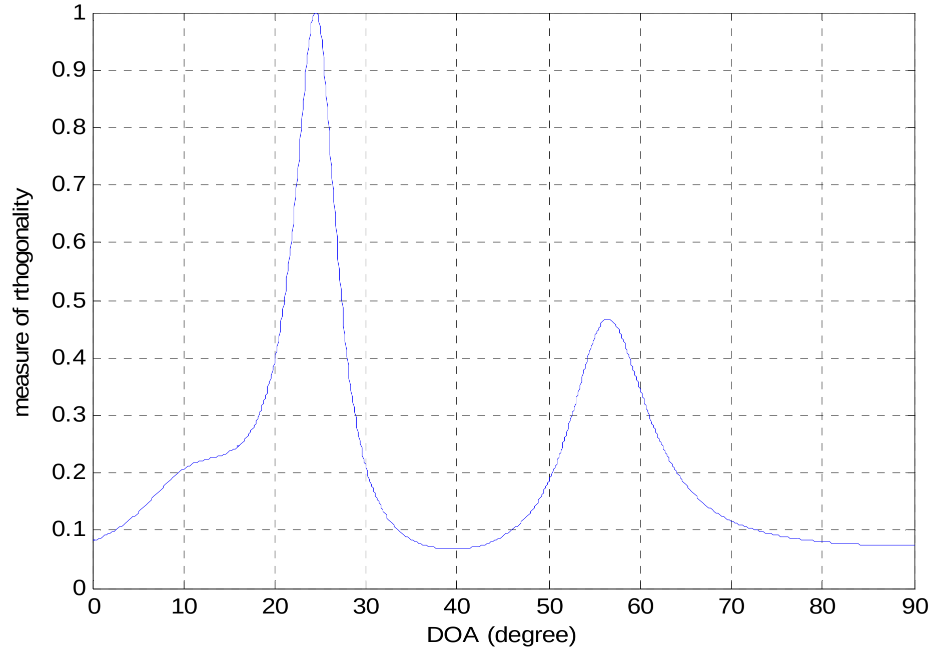

Figure (3) : Plot of the orthogonality function versus the DOA without fractional window for the 3 independent sources with SNR=-5dB, L=8 and Run=150

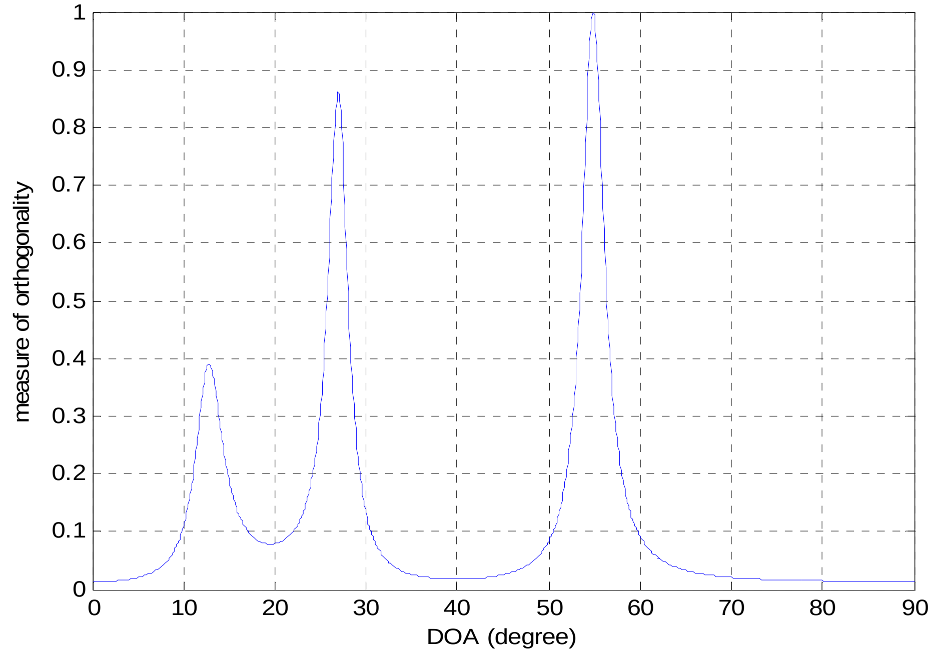

Figure (4) : Plot of the orthogonality function versus the DOA with fractional window for the 3 independent sources with α=0.9, SNR=-5dB, L=8 and Run=150

Figures (5) and (6) represent also the orthogonality function versus the DOA for an SNR=5dB without and with the fractional window respectively. The same conclusions can be made as above for the peaks of the orthogonality function.

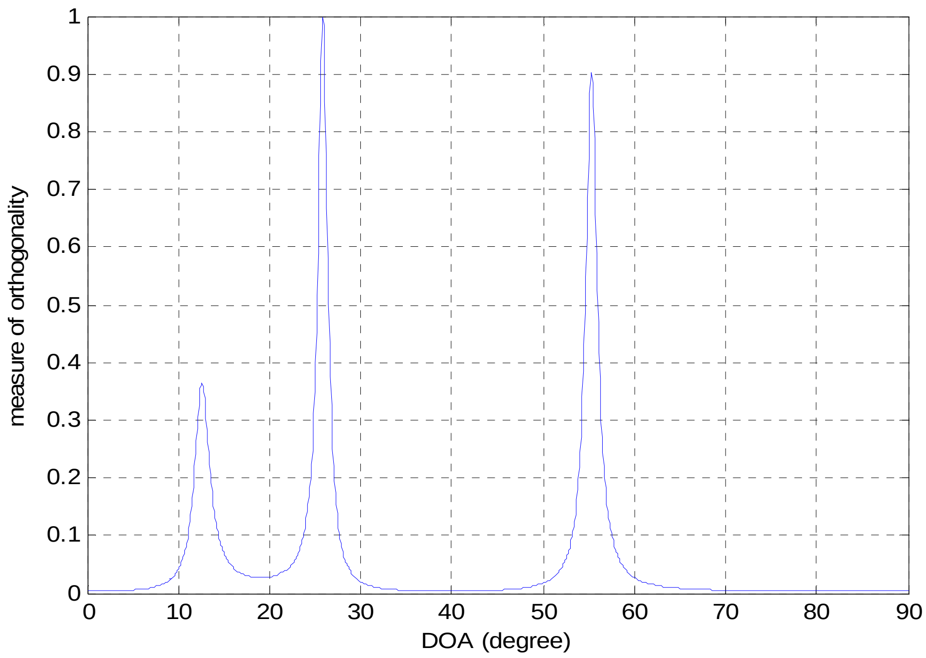

Figure (5) : Plot of the orthogonality function versus the DOA without fractional window for the 3 independent sources with SNR=5dB, L=8 and Run=150.

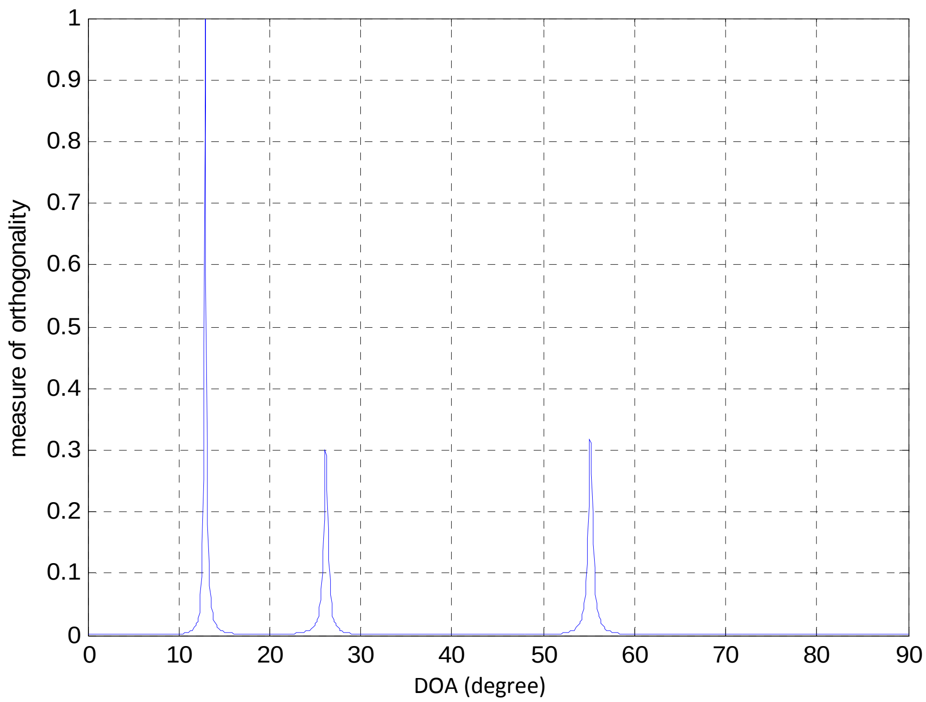

Figure (6): Plot of the orthogonality function versus the DOA with fractional window for the 3 independent sources with α=0.9, SNR=5dB, L=8 and Run=150

In figure (7) the RMSE of the DOA estimation is plotted versus the SNR with and without the fractional window. As it can be seen there is an important reduction of the RMSE of the DOA estimation for the case with the fractional window, especially in the low favorable SNR. This result is of capital importance because it is known that whenever the SNR is unfavorable the performances of the DOA estimation algorithms deteriorate and require corrections. Then by using the fractional window for the truncation of the received signal an improvement of the DOA estimation has been brought without any kind of corrections.

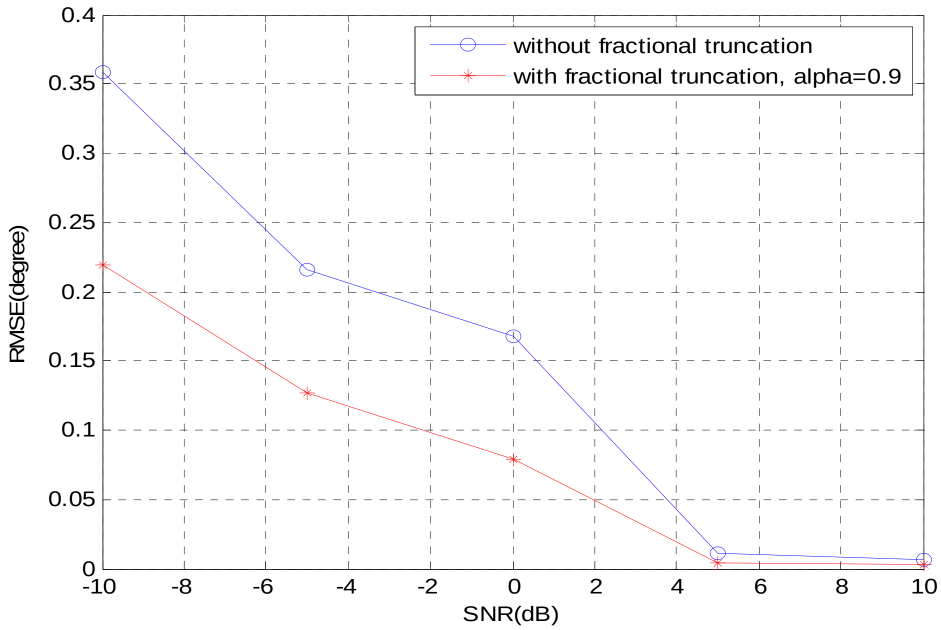

Figure (7): Plot of the RMSE of the DOA estimation versus the SNR without and with fractional window for the 3 independent sources with α=0.9, L=8, M=150 and Run = 150.

The plot of the RMSE of the DOA estimation versus the number of the observed samples M for an SNR=0 dB with and without the fractional window is shown in figure (8). An important amelioration of the RMSE of the DOA estimation can be clearly seen, especially in the region of fewer samples. This result has also its importance because we know that for fewer samples there is a serious degradation in the DOA estimation algorithms.

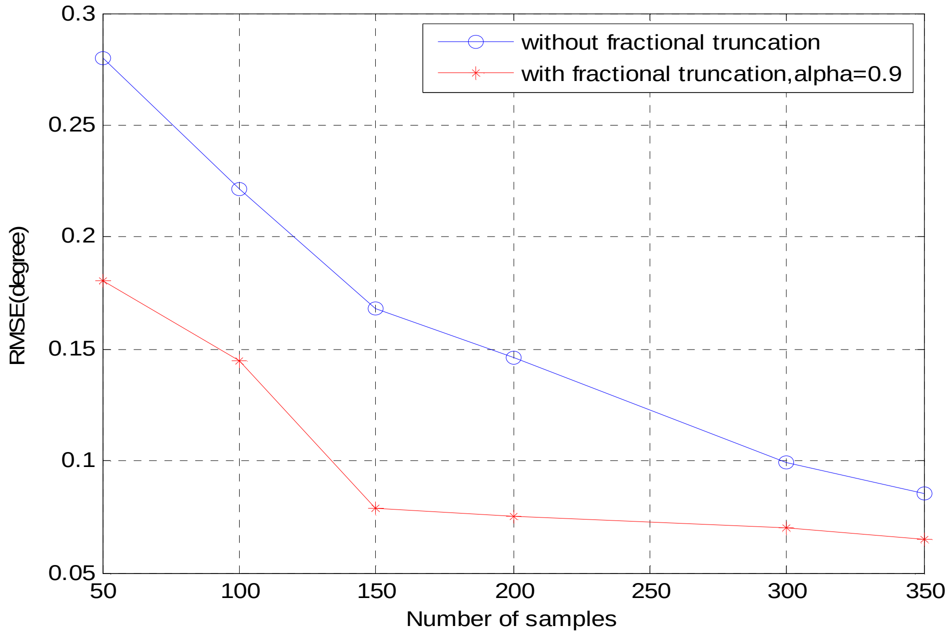

Figure (8) : Plot of the RMSE of the DOA estimation versus the M the number of observed samples without and with fractional window for the 3 independent sources with α=0.9, L=8, SNR = 0 dB and Run = 150.

Figures (9) and (10) represent the plots of the RMSE of the DOA estimation versus the angle of incidence of one source for a favorable SNR= 10 dB, without and with the fractional window respectively. In this case the angle of incidence of he source varies from 1° to 90° with a step of 1° and the field of vision is defined where the RMSE of the DOA estimation error is less than a threshold of 0.1°. Notice from figures (9) and (10) that an improvement of about 7° in the field of vision of MUSIC algorithm has also been obtained by using the fractional window

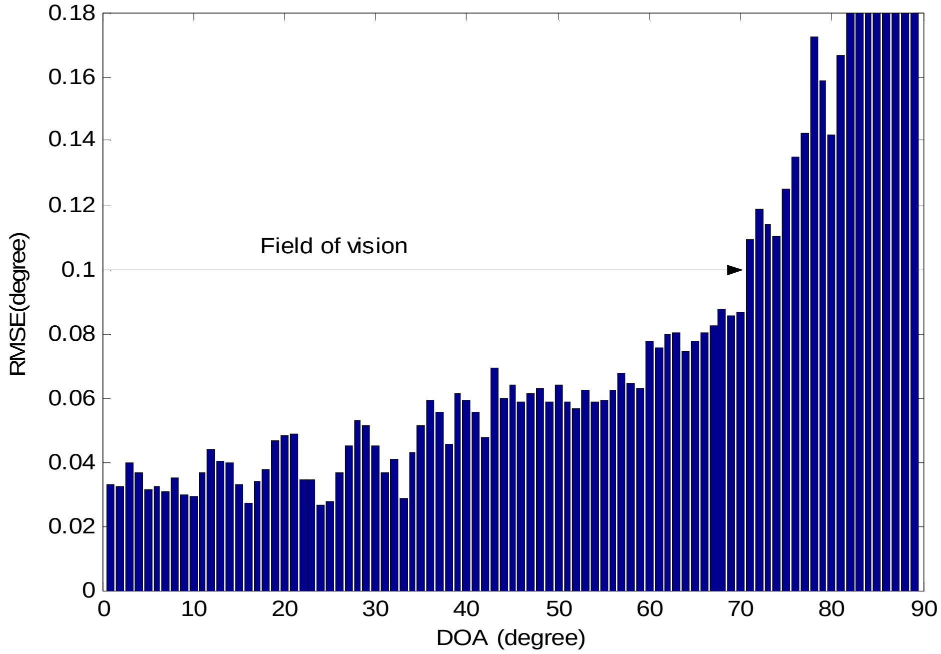

Figure (9) : Plot of the RMSE of the DOA estimation versus the angle of incidence of one source without fractional window with SNR=10dB, L=8, M=150 and Run=150

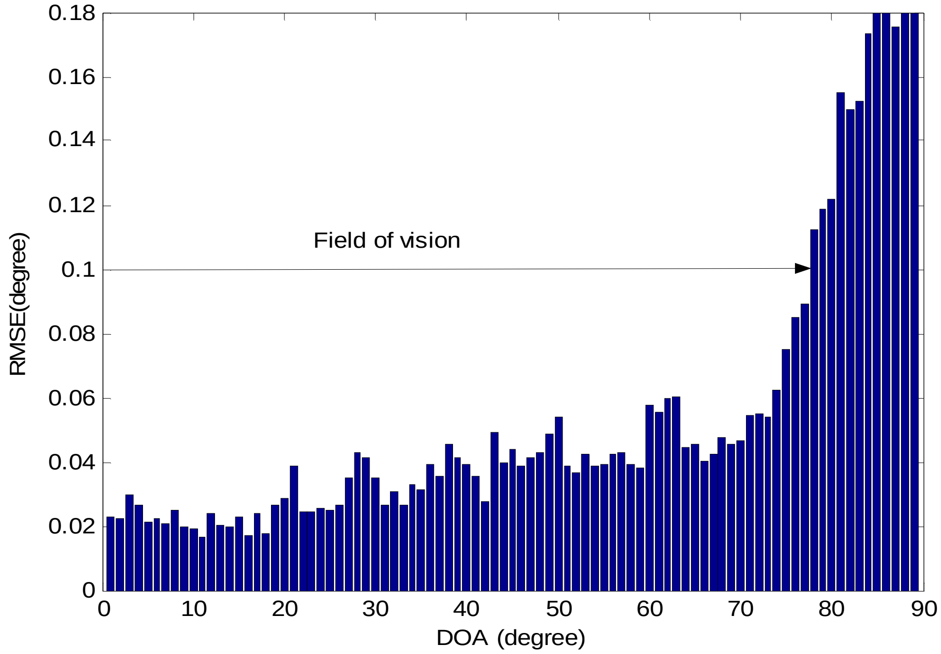

Figure (10) : Plot of the RMSE of the DOA estimation versus the angle of incidence of one source with fractional window with α=0.9, SNR=10dB, L=8, M=150 and Run=150

For an SNR=-5 dB the MUSIC algorithm detects two sources if the difference of their DOA $\Delta\theta \geq 5°$, but with the fractional window it detects two sources for $\Delta\theta \geq 2°$ only. Then the use of the fractional window for the truncation improves the resolving power of the detection for a fixed SNR. The results obtained are reported in Table1.

Table 1 : Results of the DOA estimation of two independent sources separated by $\Delta\theta$ degrees without and with fractional window with α=0.9 for L=8, SNR=-5 dB, M=150 and Run=150

$\Delta\theta$ (degree)	1	2	3	4	5	6	7	8	9	10
Number of estimated sources without fractional window	1	1	1	1	**2**	**2**	**2**	**2**	**2**	**2**
Number of estimated sources with fractional window	1	1	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**

Tables 2 and 3, summarize the use of the AIC and MDL criteria for estimating the 03 sources quoted above for unfavorable SNR=-15dB and SNR=-10dB without and with the fractional window respectively. In the first case and for both unfavorable SNR the two criteria make a false estimation, AIC criterion detects 04 sources while no source was detected by MDL criterion (table 2). However, for the case with the fractional window and for both unfavorable SNR, the two criteria AIC and MDL make an exact estimation and the 03 sources were detected (table 3).

Table 2: The number of sources estimated by AIC and MDL criteria, without Fractional window

SNR(dB) \ k		**0**	1	2	3	**4**	5	6	7	Estimated number
-15	AIC(k) x10^2	1.20	1.27	1.52	1.98	**1.09**	1.79	1.20	1.26	**4**
	MDL(k)x10^2	**1.20**	1.38	1.73	2.04	2.15	1.49	1.64	1.72	**0**
-10	AIC(k)x10^2	1.16	1.35	1.60	1.77	**1.09**	1.80	1.20	1.26	**4**
	MDL(k)x10^2	**1.16**	1.46	1.81	2.07	2.19	1.50	1.64	1.72	**0**

Table 3: The number of sources estimated by AIC and MDL criteria, with Fractional window

SNR(dB) \ k		0	1	2	**3**	4	5	6	7	Estimated number
-15	AIC(k) x10^2	2.43	2.28	2.52	**1.07**	2.73	1.08	1.17	1.26	**3**
	MDL(k)x10^2	1.24	1.40	1.75	**1.21**	2.20	1.49	1.63	1.72	**3**
-10	AIC(k)x10^2	2.29	2.36	2.60	**1.05**	2.66	1.10	1.20	1.26	**3**
	MDL(k)x10^2	1.15	1.44	1.78	**1.14**	2.16	1.48	1.64	1.72	**3**

IV. Conclusion

In this work, it has been proved that using the fractional window presented in section II in the truncation of the received signal improve the DOA estimation of D targets by L sensors array, where an important reduction in the RMSE of DOA estimation has been noted. This improvement is more important for the unfavorable SNR and fewer numbers of samples. Furthermore, the field of vision and the resolving power of the algorithm have been also increased. On the other hand, the fractional truncation remedy to the problem of false estimation noted for AIC and MDL criteria for SNR=-15dB and SNR=-10dB.

It is important to note that the dimensions of the vector of observation and the matrix of correlation have been preserved, thus the computing time and the initial memory capacity was not affected by the fractional truncation.

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