

Public Key Encryption with Keyword Search from Lattices

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Abstract. Lattice-based cryptography had recently acquired much attention due to many potential advantages: their resistance so far to cryptanalysis by quantum algorithms, their asymptotic efficiency and conceptual simplicity, and the guarantee that their random instances are as hard as the hardness of lattice problems in worst case. In this paper, we propose a public key encryption with keyword search (PEKS) using lattices. PEKS is a mechanism for searching on encrypted data. It enables user Alice to send a secret value T_w to a server that will enable the server to locate all encrypted messages containing the keyword w , but learn nothing else. The scheme can be proven secure with the hardness of the standard Learning With Errors (LWE) problem in the random oracle model.

Keywords: lattice-based cryptography, public key encryption with keyword search, provable secure.

1. Introduction

Recently, lattices have emerged as a powerful mathematical platform on which to build a rich variety of cryptographic primitives. Starting from the work of Ajtai^[1], many new constructions with lattices have been proposed, such as one-way functions and hash functions^[1,2], trapdoor functions^[3], public-key encryption schemes^[4], ID-based encryption schemes^[5-7], fully homomorphic encryption schemes^[8,9], and so on. Lattice-based schemes are attractive due to many potential advantages: their asymptotic efficiency and conceptual simplicity; their resistance so far to cryptanalysis by quantum algorithms (as opposed to schemes based on factoring or discrete log); and the guarantee that their random instances are as hard as the hardness of lattice problems in worst case. X. Boyen^[10] said that “*all those factors conspire to make lattices a prime choice, if not the primary one yet, for mathematical crypto design looking out into the future.*”

Keyword-based search technique allows users to selectively retrieve files of interest and has been widely applied in plaintext search scenarios, such as Google search. However, for privacy, data encryption may also demands the protection of keyword since keywords usually contain important information related to the data files. This leads to the traditional plaintext search techniques useless in this scenario. To securely search over encrypted data, some searchable encryption techniques have been developed^[11-18].

Public-key encryption with keyword search (PEKS) was first proposed by Boneh et.al^[12] in 2004. The mechanism enables one to search encrypted keywords without compromising the security of the original data. Suppose Bob wants to send Alice a message m with keywords w_1, \dots, w_n . Let pk_A be Alice's public key. Bob encrypts

m using a standard public key encryption $E(\cdot)$ and appends to the resulting ciphertext a list of PEKS ciphertext of each keyword. That is $E(M, pk_A) \parallel PEKS(w_1, pk_A) \parallel \dots \parallel PEKS(w_n, pk_A)$. This kind of encrypted messages may be stored in a server. Alice can give the server a certain trapdoor T_w through a secure channel that enables the server to test whether one of the keywords associated with the message is equal to the word w of Alice's choice. Given $PEKS(w', pk_A)$ and T_w , the server can test whether $w = w'$. If $w \neq w'$ the server learns nothing more about w' . PEKS mechanism can be widely used in many practical applications, such as encrypted emails extraction^[12], encrypted and searchable audit logs^[13], and encrypted files extraction in Cloud^[16].

We should note that the proposed PEKS schemes are mainly using pairings^[12,17] and some traditional cryptography tools (such as RSA)^[11]. In this paper, we propose a public-key encryption with keyword search scheme using lattices. The basic construction only gets probabilistic consistency, so we further propose a method to make up the shortfall. The scheme can be proven secure with the hardness of the standard Learning With Errors (LWE) problem in the random oracle model.

The rest of this paper is organized as follows: In Section 2, we recall some preliminary works. In Section 3 and 4, we describe the details of our new schemes and their security proofs. Finally, we conclude in Section 5.

2. Preliminaries

2.1 Public-key Encryption with Keyword Search

Recall that a PEKS scheme consists of four polynomial-time algorithms^[12]: *KeyGen*, *PEKS*, *Trapdoor*, and *Test*. The *KeyGen* algorithm generates a public/private key pair (pk, sk) . The *PEKS* algorithm produces a searchable encryption of keyword w with receiver's public key. The *Trapdoor* algorithm generates a trapdoor T_w for keyword w with receiver's private key, and the *Test* algorithm verifies whether a cipher-text matches a trapdoor.

The general notion of security of PEKS scheme is indistinguishability against chosen keyword attack^[12]. A PEKS scheme is semantic security if no polynomial time adversary A has a non-negligible advantage against a challenger C in the following game.

Security Game:

KeyGen. C runs *KeyGen* algorithm to generate a key pair (pk, sk) , and gives pk to A .

Phase 1. A can adaptively ask the challenger for the trapdoor T_w for any keyword $w \in \{0,1\}^*$ of his choice.

Challenge. At some point, A sends the challenger two words w_0, w_1 on which it wishes to be challenged. The only restriction is that A did not previously ask for the

trapdoors T_{w_0} or T_{w_1} . The challenger picks a random $b \in \{0,1\}$ and gives the attacker $C = PEKS(pk, w_b)$ as the challenge PEKS cipher-text.

Phase 2. A can continue to adaptively ask the challenger for the trapdoor T_w for any keyword w of his choice as long as $w \neq w_0, w_1$.

Guess. Eventually, the attacker A outputs $b' \in \{0,1\}$ and wins the game if $b = b'$.

The advantage of A in this game is defined as $|\Pr[b' = b] - \frac{1}{2}|$.

2.2 Lattices and hardness assumption

Definition 1. Given n linearly independent vectors $b_1, b_2, \dots, b_n \in R^m$, the lattice Λ generated by them is denoted $L(b_1, b_2, \dots, b_n)$ and define as:

$$L(b_1, b_2, \dots, b_n) = \left\{ \sum_{i=1}^m x_i b_i \mid x_i \in Z \right\}$$

The vectors b_1, b_2, \dots, b_n are called the basis of the lattice. Let $B = \{b_1, b_2, \dots, b_n\}$, we let \tilde{B} denote its Gram-Schmidt orthogonalization of the vectors b_1, b_2, \dots, b_n taken in that order, and we let $\|S\|$ denote the length of the longest vector in B for the Euclidean norm.

Definition 2. For q prime and $A \in Z_q^{n \times m}$ and $u \in Z_q^n$, define:

$$\Lambda_q^\perp(A) = \{e \in Z^m \mid A \cdot e = 0 \pmod{q}\}$$

$$\Lambda_q^u(A) = \{e \in Z^m \mid A \cdot e = u \pmod{q}\}$$

Ajtai^[1] and later Alwen and Peikert^[19] showed how to sample an essentially uniform matrix $A \in Z_q^{n \times m}$ with an associated basis T_A of $\Lambda_q^\perp(A)$ with low Gram-Schmidt norm.

Proposition 1^[20]. For any prime $q \geq 2$ and $m \geq 5n \log q$, there exists a probabilistic polynomial-time algorithm *TrapGen* that outputs a pair $A \in Z_q^{n \times m}$ and T_A such that A is statistically close to uniform and T_A is a basis for $\Lambda_q^\perp(A)$ with length $L = \|\tilde{T}_A\| \leq m\omega(\sqrt{\log m})$ with all but $n^{-\omega(1)}$ probability.

We further review Gaussian functions used in lattice based cryptographic constructions.

Definition 3. Let m be a positive integer and $\Lambda \in R^m$ be a m dimensional lattice. For any vector $c \in R^m$ and any positive parameter $\sigma \in R_{>0}$, we define:

$$\rho_{\sigma,c}(x) = \exp(-\pi \|x - c\|^2 / \sigma^2) \quad \text{and} \quad \rho_{\sigma,c}(\Lambda) = \sum_{x \in \Lambda} \rho_{\sigma,c}(x)$$

The discrete Gaussian distribution over Λ with center c and parameter σ is

$$\forall y \in \Lambda \quad D_{\Lambda, \sigma, c}(y) = \frac{\rho_{\sigma, c}(y)}{\rho_{\sigma, c}(\Lambda)}$$

For notational convenience, $\rho_{\Lambda, \sigma, 0}$ and $D_{\Lambda, \sigma, 0}$ are abbreviated as ρ_{σ} and $D_{\Lambda, \sigma}$.

Gentry *et al.* [3] construct the following algorithm for sampling from the discrete Gaussian $D_{\Lambda, \sigma, c}$, given a basis T_A for the m -dimensional lattice Λ with $\sigma \geq \|\tilde{T}_A\| \omega(\sqrt{\log m})$. Specialized to the case of random lattices, they show an algorithm:

SamplePre(A, T_A, u, σ): On input a matrix $A \in Z_q^{n \times m}$ with ‘short’ trapdoor basis T_A for $\Lambda_q^\perp(A)$, a target image $u \in Z_q^n$ and a Gaussian parameter $\sigma \geq \|\tilde{T}_A\| \omega(\sqrt{\log m})$, outputs a sample $e \in Z_q^m$ from a distribution that is within negligible statistical distance of $D_{\Lambda_q^u(A), \sigma}$.

Security of all our construction reduces to the LWE problem, a classic hard problem on lattices defined by Regev [4].

Definition 4. Consider a prime q , a positive integer n , and a distribution χ over Z_q , all public. An (Z_q, n, χ) -LWE problem instance consists of access to an unspecified challenge oracle Ψ , being either, a noisy pseudo-random sampler Ψ_s carrying some constant random secret key $s \in Z_q^n$, or, a truly random sampler Ψ_ϕ , whose behaviors are respectively as follows:

-- Ψ_s : output noisy pseudo-random sample of the form $(u_i, v_i) = (u_i, u_i^T \cdot s + x_i) \in Z_q^n \times Z_q$, where $s \in Z_q^n$ is a uniformly distributed persistent value invariant across invocations, $x_i \in Z_q$ is a fresh sample from χ , and u_i is uniform in Z_q^n .

-- Ψ_ϕ : output truly uniform random sample from $Z_q^n \times Z_q$.

The (Z_q, n, χ) -LWE problem allows repeated queries to the challenge oracle Ψ .

We say that an algorithm A decides the (Z_q, n, χ) -LWE problem if

$|\Pr[A^{\Psi_s} = 1] - \Pr[A^{\Psi_\phi} = 1]|$ is non-negligible for a random $s \in Z_q^n$.

Regev has showed that for certain noise distributions χ , the LWE problem is as hard as the worst-case SIVP and GapsSVP under a quantum reduction.

Proposition 2^[4]. For an $\alpha \in (0,1)$ and a prime $q > 2\sqrt{n}/\alpha$, let $\bar{\psi}_\alpha$ denote the distribution over Z_q of the random variable $\lfloor qX + \frac{1}{2} \rfloor \bmod q$ where X is a normal random variable with mean 0 and standard deviation $\alpha/\sqrt{2\pi}$. Then, if there exists an efficient, possibly quantum, algorithm for deciding the (Z_q, n, χ) -LWE problem, there exists a quantum poly-time algorithm for approximating the SIVP and GapSVP problems, to within $\tilde{O}(n/\alpha)$ factors in the l_2 norm, in the worst case.

3. A PEKS Scheme from Lattice

3.1 The Basic Construction

Let n, m be positive integers, q be prime, with $q \geq 2$ and $m \geq 5n \log q$, $H : \{0,1\}^* \rightarrow Z_q^n$ be a hash function. The scheme is described as follows:

- **KeyGen(1^n)**: On input a security parameter n , invoke $TrapGen(q,n)$ to generate a uniformly random matrix $A \in Z_q^{n \times m}$ along with a short basis T_A for $\Lambda_q^\perp(A)$, and output the public key $pk = A$ and secret key $sk = T_A$.
- **PEKS(A, w)**: Compute $u = H(w)$, select $s \leftarrow Z_q^n$ uniformly and compute $p = A^T \cdot s + x \in Z_q^m$ and $c = u^T \cdot s + y \in Z_q$, where $x \leftarrow \chi^m$ and $y \leftarrow \chi$ are noise vectors. Output the cipher-text (p,c) .
- **Trapdoor(A, T_A, w)**: Compute $u = H(w)$ and choose a $e \in Z_q^m$ satisfying $A \cdot e = u$ by $e \leftarrow SamplePr e(A, T_A, u, \sigma)$, where σ is a Gaussian parameter. Output e as the trapdoor.
- **Test($(p,c), e$)**: Compute $b = c - e^T \cdot p$. If $|b| < q/4$, output 1 (“yes”); otherwise, output 0 (“no”).

Suppose $(p,c) = PEKS(A, w)$ and $e = Trapdoor(A, T_A, w')$, then

$$b = c - e^T p = u^T \cdot s + y - e^T (A^T \cdot s + x) = (u^T - (A \cdot e)^T) \cdot s + y - e^T \cdot x$$

It is easy to see that if $w = w'$, then $|b| = |y - e^T \cdot x|$, just as discussion in [3], with parameters $q \geq 5rm$, $\alpha \leq 1/(r\sqrt{m} \cdot \omega(\sqrt{\log n}))$, and $\chi = \overline{\Psi}_\alpha$, with overwhelming probability we have $|b| < q/4$, where r is a Gaussian parameter.

However, if $w \neq w'$, with the randomness of u and e , $u^T - (A \cdot e)^T$ can be seen as a random vector in Z_q^n . That is $|b|$ is a random in $[0, q]$ (over the random choices of u and e). Hence, the probability of $|b| < q/4$ is $\frac{1}{4}$. This means that there is probability $\frac{1}{4}$ that the *Test* algorithm output 1 when $w \neq w'$, and so the consistency of the scheme can not be ensured. We will deal with this problem in the following subsection.

3.2 Improvement for consistency

In order to decrease the possibility that the *Test* algorithm output 1 when $w \neq w'$, we can modified the basic construction as follows.

Let $k = \text{poly}(n)$, for $i = 1, \dots, k$, $H_i : \{0,1\}^* \rightarrow Z_q^n$ be k different hash functions. The *KeyGen* algorithm is the same as before.

- *PEKS*(A, w): Select $s \leftarrow Z_q^n$ uniformly and compute $p = A^T \cdot s + x \in Z_q^m$. For $i = 1, \dots, k$, compute $u_i = H_i(w)$, and $c_i = u_i^T \cdot s + y_i \in Z_q$, where $x \leftarrow \chi^m$ and $y_i \leftarrow \chi$ are noise vectors. Output the cipher-text $(p, \{c_i\}_{i=1, \dots, k})$.
- *Trapdoor*(A, T_A, w): For $i = 1, \dots, k$, compute $u_i = H_i(w)$ and choose a $e_i \in Z_q^m$ satisfying $A \cdot e_i = u_i$ by $e_i \leftarrow \text{SamplePre}(A, T_A, u_i, \sigma)$, where σ is a Gaussian parameter. Output $(\{e_i\}_{i=1, \dots, k})$ as the trapdoor.
- *Test*($(p, \{c_i\}_{i=1, \dots, k}), \{e_i\}_{i=1, \dots, k}$): For $i = 1, \dots, k$, compute $b_i = c_i - e_i^T \cdot p$. If for all $i = 1, \dots, k$, $|b_i| < q/4$, output 1 (“yes”); otherwise, output 0 (“no”).

Just as above discussion, if $w = w'$, then the *Test* algorithm output 1 with overwhelming probability. And if $w \neq w'$, the *Test* algorithm output 0 with probability $1 - 2^{-2k}$. When k is large enough, we can ensure the consistency of the scheme.

3.3 Efficiency analysis

To guarantee the security from LWE problem, we select the parameters of our scheme following [21]: with the security parameter k , the public key is the matrix $A \in Z_q^{n \times m}$, where $n = k^3$, $q = 2^{\sqrt{n}} = 2^{k^{1.5}}$ and $m = 2n \log q \approx 2n\sqrt{n} = O(k^{4.5})$. We now analyze the computation of the basic construction. The computation of p and c in the *PKES* algorithm are $m \cdot n(\log q)^2 + m \log q$ and $n(\log q)^2 + \log q$ respectively. Hence the total computation is $(m+1) \cdot n(\log q)^2 + (m+1) \log q$. The computation of the *Test* algorithm is $m(\log q)^2 + \log q$.

We also implemented the algorithms of our scheme with Maple.14, on an Intel 2.9 GHz Core i7 system. With $k = 5, m = 2796, n = 125, q = 2321$, the size of public key is about 0.5MB, and the time of a *PEKS* algorithm is about 0.3 seconds.

Since its first introduction by Boneh et.al^[12], the proposed *PEKS* schemes are mainly using pairings^[12,17] and some traditional cryptography tools (such as RSA)^[11]. Recently, due to the potential advantages, many lattices-based public key encryption schemes have been proposed^[4-7]. But to the best of our knowledge, this is the first lattices-based *public-key encryption with keyword search* scheme.

3.4 Security Proof

We present the security proof for the basic construction. The security proof of improvement version is obviously almost the same.

Proposition 3. In the random oracle mode, suppose there is an polynomial-time adversary \mathcal{F} that have non-negligible advantage ε in attacking the scheme, then we can construct a polynomial-time algorithm \mathcal{S} that can solve the LWE problem with

probability $\frac{2\varepsilon(Q_H - 1)}{Q_H^2}$, where Q_H denotes the number of queries that \mathcal{F} can query

to the random oracle $H(\cdot)$.

Proof. With the adversary \mathcal{F} , we construct the algorithm \mathcal{S} as follows:

\mathcal{S} requests from Ψ and receives, for each $i = 0, \dots, m$, a fresh pair $(u_i, v_i) \in Z_q^n \times Z_q$, and selects a random integer $Q^* \in [1, Q_H]$.

KeyGen. \mathcal{S} assemble the random matrix $A \in Z_q^{n \times m}$ from m of given LWE samples by letting the i _th column of A be the vector u_i for all $i = 1, \dots, m$, and gives the public key A to \mathcal{F} .

Phase 1. \mathcal{S} answers queries of \mathcal{F} as follows:

-- $H(\cdot)$ query: For the Q -th query input $w \in \{0,1\}^*$, if this is query number Q^* (i.e. $Q = Q^*$, S defines $H(w) = u_0$ and return $H(w)$. Otherwise, S chooses $e \leftarrow D_{Z^m, r}$, and computes $u = A \cdot e$, saves the tuple (w, u, e) in H_list for future use, and returns $H(w) = u$.

-- $Trapdoor(\cdot)$ query: For input $w \in \{0,1\}^*$, if $H(w) = u_0$, S aborts and fails. Otherwise, S retrieves the saved tuple (w, u, e) from the H_list (w.l.o.g., we can assume that an $Trapdoor$ query on w is preceded by a $H(\cdot)$ query with w). By construction, e is the trapdoor for keyword w . Return e as the trapdoor to the adversary.

Challenge. Once \mathcal{F} decides that Phase 1 is over it outputs two keywords w_0, w_1 on which it wishes to be challenged. S runs the above algorithm for responding to $H(\cdot)$ queries twice with input w_0, w_1 . If both $H(w_0) \neq u_0$ and $H(w_1) \neq u_0$, then S aborts. Otherwise, suppose $H(w_b) = u_0$ S retrieves $v_0, \dots, v_m \in Z_q$ from the LWE instance, and sets $c_0 = [v_1, \dots, v_m]^T \in Z_q^m$, $c_1 = v_0 \in Z_q$. S responds with the challenge cipher-text (c_0, c_1) . (Note: When Ψ is a pseudo-random LWE oracle then $c_0 = A^T \cdot s + x$ and $c_1 = u_0^T \cdot s + y$ for some random $s \in Z_q^n$ and noise values $x \in \mathcal{X}^m$ and $y \in \mathcal{X}$. In this case (c_0, c_1) is a valid encryption for w_b . When O is a random oracle then (c_0, c_1) is uniform in $Z_q^m \times Z_q$.)

Phase 2. S answers queries of \mathcal{F} the same way it does in Phase 1 with the only restriction that $w \neq w_0, w_1$ for $Trapdoor$ queries.

Guess. Eventually \mathcal{F} outputs $b' \in \{0,1\}$.

Finally, if $b = b'$ S outputs 1, otherwise S outputs 0. The distribution of the public parameters is identical to its distribution in the real system as are responses to private key queries. If S does not abort then the challenge cipher-text is distributed either as in the real system or is independently random in $Z_q^m \times Z_q$. Hence, if S does not abort then its advantage in solving LWE is the same as \mathcal{F} 's advantage in attacking

the system.

Because Q^* is chosen randomly in 1 and Q_H , S does not abort in the simulation of $Trapdoor(\cdot)$ with probability $1-1/Q_H$. The probability that $H(w_0) = u_0$ or $H(w_1) = u_0$ is $2/Q_H$. So we conclude that the advantage of S in solving LWE is $\frac{2\epsilon(Q_H - 1)}{Q_H^2}$.

4. Conclusion

Lattice-based cryptosystems are becoming increasingly popular in the research community. In this paper, we propose a PEKS scheme using lattices. Although fruitful lattices-based cryptographic schemes have been proposed, but to the best of our knowledge, this is the first lattices-based *public-key encryption with keyword search* scheme. The scheme can be proven secure with the hardness of the standard LWE problem in the random oracle model. We should note that it may be not a difficult task to modify this scheme to obtain a version secure in the standard model.

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